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14.30 Introduction to Statistical Methods in Economics
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Problem Set #3

14.30 - Intro. to Statistical Methods in Economics

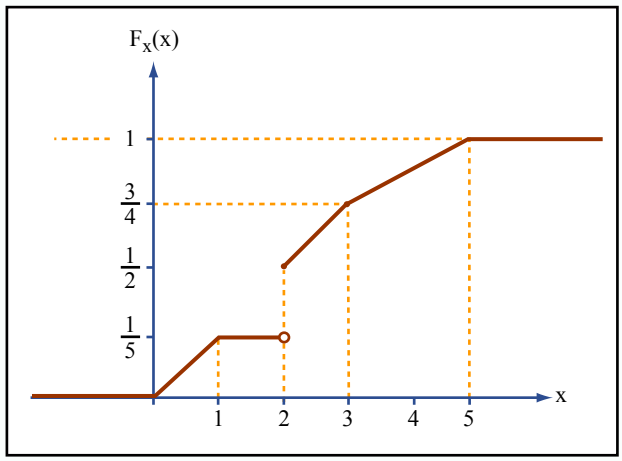
Instructor: Konrad Menzel

Due: Tuesday, March 3, 2009

Question One

1. Write down the definition of a cumulative distribution function (CDF). Explain what it means in words, perhaps using an example.

2. Verify whether the following function is a valid CDF. If yes, draw a graph of the corresponding PDF.



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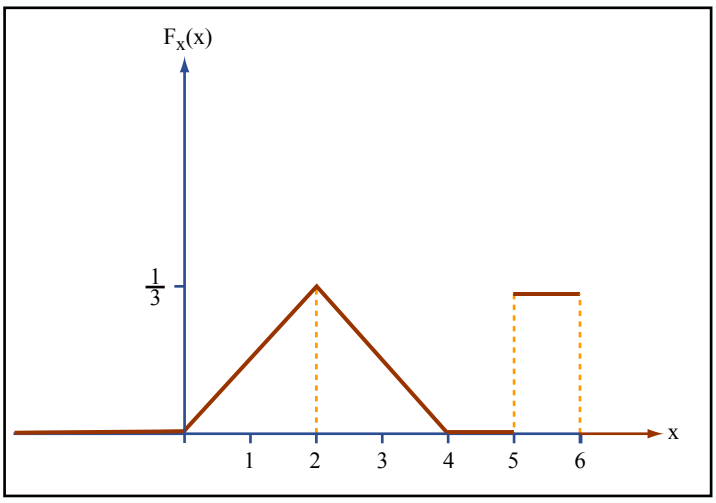
3. Verify that the following function is a valid PDF and draw the corresponding CDF.



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Question Two

1. Give a p.d.f. whose c.d.f. is not continuous but is right-continuous.

True/false/uncertain: Always give a brief explanation if the statement is true, or counter-examples and a short explanation of the counter-examples if the statement is false or uncertain.

1. If $P(A|B) > P(A)$ and $P(A|C) > P(A)$, then $P(A|B,C) > P(A)$.
2. A continuous p.d.f. can never take a value greater than 1.
3. $P(A) = P(A|B)P(B)$ means that A and B are independent.

Question Three

(Source: Bain/Engelhardt, Ch. 2, ex. 8)

A nonnegative integer-valued random variable X has a CDF of the form $F(x) = 1 - (1/2)^{x+1}$ for $x = 0, 1, 2, \ldots$ and zero if $x < 0$.

1. Find the pdf of X.
2. Find $P[10 < X \leq 20]$.
3. Find $P[X \, is \, even]$.

Question Four

1. Suppose that a random variable has a PDF that is proportional to x on the interval $[0, 1]$. Write down a formula for this PDF. What is the corresponding CDF?
2. Now suppose that the random variable has a CDF that is porportional to x on the interval $[0, 1]$. Write down a formula for this CDF. What is the corresponding PDF?

Question Five

Suppose that the joint PDF of X and Y is given by

$$f_{X,Y} = \begin{cases} kx^3y & \text{for } 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

1. What is the value of k?

2. What is the marginal PDF, $f_X(x)$, of x?

3. What is the value of the marginal cdf of x, $F_X(x)$, at $x = \frac{1}{2}$?

4. What is the conditional PDF of y (conditional on x, i.e. $f(y|x)$)? Are X and Y independent? Explain.

5. What is the probability that $X + Y < 1$?

Question Six

(Bain/Engelhardt, Ch. 2, ex. 10)

Let X be a discrete random variable such that $P[X = x] = 0$ otherwise. Suppose the CDF is $F(x) = .05x(1 + x)$ at the values $x = 1$, 2, 3, or 4.

1. Sketch the graph of the CDF.

2. Sketch the graph of the discrete pdf, $f(x)$.

3. Write down the definition of $E[X]$ and find $E[X]$.