14.121 Final Exam
October 28, 2005

Answer all questions. You have 90 minutes in which to complete the exam. Don't spend too much time on any one question.

1. (15 Minutes – 20 Points) Answer each of the following subquestions BRIEFLY.

(a) Define equivalent variation and compensating variation. Given an example of a problem for which equivalent variation would be an appropriate concept to apply.

(b) Find the Hicksian demand function for a consumer with $u(x_1, x_2) = \sqrt{x_1} + x_2$.

(c) The traditional celebration of Thanksgiving in the United States involves families gathering together and eating a large meal that includes a whole roasted turkey. The tradition is widely followed: over 95% of Thanksgiving meals include a turkey and twenty times more turkeys are sold during the Thanksgiving week than in a normal week. An initially puzzling observation is that supermarkets typically put turkeys on sale during this week. Why might this occur?

2. (20 Minutes – 27 Points)

Suppose that Glenn Ellison is considering purchasing flood insurance for his house. If Glenn does not buy flood insurance, his wealth will be w if there is no flood and $w - L$ if there is a flood. The probability of a flood is π.

The price of a policy that pays K if a flood occurs is cK. Assume that $c < 1$. (Note that the problem is otherwise uninteresting because Glenn would never buy any insurance.)

Assume that Glenn can choose any $K \in [0, L]$, and that his choice of how much insurance to buy maximizes his expected utility. Assume that Glenn's von Neumann-Morgenstern utility function u is a differentiable, strictly increasing and concave function of his final wealth, i.e. Glenn maximizes $(1 - \pi)u(w - cK) + \pi u(w - L - cK + K)$.

(a) Find the first-order condition that characterizes Glenn's choice of K (assuming that the parameters are such that an interior optimum exists.)

(b)For what value(s) of c will Glenn purchase full insurance? Does the answer depend on the form of the utility function u? Why is this?

(c) Drop the assumptions that u is differentiable and concave – assume only that u is strictly increasing and that a utility-maximizing choice exists. Show that the K that Glenn chooses is weakly increasing in the probability π of a flood occurring.

3. (25 Minutes – 30 Points)

Consider an economy with three goods. Suppose that a consumer has a continuous utility function satisfying local nonsatiation. Suppose also that the consumer's Walrasian demands for goods 1 and 2 when $p_3 = 1$ satisfy

$$\begin{aligned} x_1(p_1, p_2, 1, W) &= a_1 + b_1 p_1 + c_1 p_2 + d_1 p_1 p_2 \\ x_2(p_1, p_2, 1, W) &= a_2 + b_2 p_1 + c_2 p_2 + d_2 p_1 p_2 \end{aligned}$$

(a) State Walras' law and use it to find the Walrasian demand for good 3. (It's fine to just give the demand when $p_3 = 1$.)

(b) State a result about the homogeneity of Walrasian demands and use it to find the consumer's Walrasian demands at other values of p_3.

(c) Note that the Walrasian demands for goods 1 and 2 are independent of wealth. Show that this makes it very easy to find the Hicksian demands for goods 1 and 2. State the Compensated Law of Demand. Show that this law puts some restrictions on the possible values for $(a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2)$.

(d) Define the Slutsky substitution matrix. What properties must it have if demands are derived from maximizing a continuous, locally nonsatiated, and strictly quasiconcave utility function? Give at least one additional restriction on $(a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2)$ that this implies.

4. (25 Minutes – 23 points)

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of states. Let $\Delta(X)$ be the set of lotteries with outcomes in X. Write δ_{x_i} for the lottery in which x_i is realized with probability one.

Let $\succcurlyeq_P$ be a preference on $\Delta(X)$. Assume that $\succcurlyeq_P$ is transitive and that $\delta_{x_1} \succcurlyeq_P \delta_{x_2} \succcurlyeq_P \ldots \succcurlyeq_P \delta_{x_n}$.

(a) State the Archimedian axiom.

(b) The preference $\succcurlyeq_P$ is said to satisfy *monotonicity* if $a\delta_{x_1} + (1-a)\delta_{x_n} \succcurlyeq_P b\delta_{x_1} + (1-b)\delta_{x_n}$ if and only if $a \geq b$. Show that monotonicity implies that $\delta_{x_1} \succ_P \delta_{x_n}$.

(c) The preference $\succcurlyeq_P$ is said to satisfy *continuity* if for all $x \in \Delta(X)$ there exists an $a \in [0, 1]$ such that $a\delta_{x_1} + (1-a)\delta_{x_n} \sim_P x$. Show that if $\succcurlyeq_P$ satisfies monotonicity and continuity, then it satisfies the Archimedian axiom.