Econ 14.04 Fall 2006
Assignment 3: Slutsky Matrix, Revealed Preference, Aggregation, and Uncertainty

The due date for this assignment is Monday October 16th.
Reading assignment: Sections 8-9.

1. Suppose that a consumer with utility U(x$_1, x_2$) is given an iniitial endowment $\overline{x}_1, \overline{x}_2$ so that his total budget set is $p_1\overline{x}_1, p_2\overline{x}_2$

   (a) Derive a formula for the slutsky equation with endowments

   (b) Suppose that p_1, p_2 are such that consumption is exactly equal to the initial endowment. Show that $\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}$ in this special case.

2. You are given the following partial information about a consumer's purchases. he consumes only two goods.:

	Year 1 Quantity	Price		Year 2 Quantity	Price
Good 1	100	100	Good 1	120	100
Good 2	100	100	Good 2	?	80

   Over what range of quantitites of good 2 consumed in year 2 would you conclude:

   (a) That his behaviour is inconsistent (i.e. it contradicts the weak axiom of revealed preference)

   (b) That the consumer's consumption bundle in year 1 is revealed preferred to that in year 2?

   (c) That the conumer's consumption bundle in year 2 is revealed preferred to that in year 1?

   (d) That there is insufficient information to justify (a),(b),(c)

   (e) *That good 1 is an inferior good (at some price) for this consumer? Assume that the weak axiom is satisfied

   (f) *That good 2 is an inferior good (at some price) for this consumer? Assume that the weak axiom is satisfied.

3. Recall that for demand functions to come from a utility maximization function:

   - They must be homogeneous of degree 0. That is $x(tp_1, tp_2, tm) = x(p_1, p_2, m)$
   - They must satisfy the budget constraint $p \cdot x = m$
   - The slutsky matrix must be symmetric. $(\frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial m}x_1 = \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial m}x_2)$
   - The slutsky matrix is equal to the matrix of hicksian demand functions:

$$\begin{bmatrix} \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m}x_1 & \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial m}x_2 \\ \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial m}x_1 & \frac{\partial x_2}{\partial p_2} + \frac{\partial x_2}{\partial m}x_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1}{\partial p_1} & \frac{\partial h_1}{\partial p_2} \\ \frac{\partial h_2}{\partial p_1} & \frac{\partial h_2}{\partial p_2} \end{bmatrix}$$

(a) Suppose that we have 2 goods and consider the demand functions x(p,m) defined by:

$$
\begin{aligned}
x_1(p,m) &= \frac{p_2}{p_1+p_2}\frac{m^\alpha}{p_1} \\
x_2(p,m) &= \frac{\beta p_1}{p_1+p_2}\frac{m}{p_2}
\end{aligned}
$$

Determine α and β

(b) *Suppose that we have 2 goods and the hicksian demand function for good one $h_1(p,u)$:

$$h_1(\mathbf{p},u) = \frac{p_2}{p_1}u$$

1. Given that $\frac{\partial h_1}{\partial p_2} = \frac{\partial h_2}{\partial p_1}$, show that for $p_2 > 0$, $h_2(\mathbf{p},u) = \ln\left(\frac{p_1}{p_2}\right)u + g(p_2,u)$ where g is an arbitrary function.
2. Suppose that when $p_1 = p_2$, $h_1(\mathbf{p},u) = u, h_2(\mathbf{p},u) = 0$. Determine $h_2(\mathbf{p},u)$.

4. Consider the utility function $\ln(x) + y$.

(a) Optional: Solve for $x(\mathbf{p},m), y(\mathbf{p},m), v(\mathbf{p},m), e(\mathbf{p},u), x^h(\mathbf{p},m), y^h(\mathbf{p},m)$. Follow the solutions from PS2 if necessary.

(b) Suppose that there are 5 people in the economy each with endowments m^i, $i = 1,2,3,4,5$.

1. Suppose that $m^i > p_y \ \forall i$. Construct the aggregate demand function for x and y.What properties do the individual demand functions have that simplifty this problem?
2. Now suppose that $m^1 < p_y$, $m^2 < p_y, m^i > p_y$ for $i = 3,4,5$. Construct the aggregate demand for x_1,x_2

5. Suppose that an agent is strictly risk-averse who has an initial wealth of w but who runs a risk of a loss of D dollars. The probability of loss is π. It is possible, however, for the decision maker to buy insurance. One unit of insurance costs q dollars and pays 1 dollar if the loss occurs. Thus, if α units of insurance are bought, the wealth of the individual will be $w - \alpha q$ if there is no loss and $w - \alpha q - D + \alpha$ if the loss occurs. The utility maximizer thus solves:

$$\max_{\alpha \geq 0}(1-\pi)u(w-\alpha q) + \pi u(w - \alpha q - D + \alpha)$$

(a) Assume that at the optimum $\alpha > 0$. Find the FOC for the problem.

(b) Suppose that insurance is actuarially fair in the sense of it being equal to the expected cost of insurance. Find α^*

(c) **Optional: Suppose that the agent is insuring his car and once a week he goes to white castle for burgers. There is no drive through and his car must be pushed in order for him to restart it which requires him some effort e. If he turns off the car (and thus expends effort) he has a probability $\pi(e)$ of his car being stolen while in the restaurant. if he does not spend the effort he has a probability of $\pi(0)$ of his car being stolen where $\pi(0) > \pi(e)$.

1. Suppose that he is able to get the amount of insurance as in part (b) above. Show that he will always choose to leave his car running.
2. Let $u(x) = \ln(x)$. If the agent lets his car run while in white castle his utility is:

$$[1 - \pi(0)] \ln(w - \alpha q) + \pi(0) \ln(w - \alpha q - D + \alpha)$$

   If the agent turns off his car his utility is:

$$[1 - \pi(e)] \ln(w - \alpha q) + \pi(e) \ln(w - \alpha q - D + \alpha) - e$$

   Show that for the agent to choose to turn off his car, $\alpha^{SB} < D$.
3. Draw the function $ln(x)$. Why is the agents utility lower in part ii than in part b?