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**GREG HUTKO:** Hi, and welcome back to the 14.01 problem-solving videos. Today I'm going to do Fall 2010 Problem Set 7, Problem Number 2. And we're going to work through both parts A and B to begin with, so I'm going to read the beginning part of the problem.

Suppose Napster's considering selling music via email. There are two types of users, students and non-students. Each non-student has an inverse demand function of  $200 - x$  and each student has an inverse demand function of  $160 - x$ , where  $x$  is the number of songs delivered by email,  $p$  is measured in cents. The marginal cost to Napster of sending an additional song via email is 0. Suppose Napster can identify all users as either students or non-students. If Napster offers a fixed number of songs per year to each person, what is the profit maximizing level of songs offered to a student and a non-student. In other words, what is the equilibrium level of output for each type of person under first degree price discrimination?

Part B is going to ask us, given the equilibrium level of outputs that we've calculated, what is the dollar price charged to students and non-students per year?

Now what this problem is really asking us about, it's asking us about a situation in economics called a two-part tariff. Now in some cases, when you have two groups of consumers that have differing demand functions, a monopolist can actually capture the vast majority of the consumer surplus. Or in most cases, all the consumer surplus if he can differentiate between the two groups. So in this case, Napster is probably going to set up a situation where they can differentiate between students and non-students. So if they can discriminate by a student by having them enter their school ID or their school email address, then they'd be able to do the situation that we're doing for parts A and B.

The two parts of a two-part tariff, the first part is an access fee that's equal to the consumer surplus. So you can say to a consumer, sure, you can have this set bundle of songs, this set number of songs. But you're going to have to pay all of the potential benefit that it would bring you. So if it's going to bring you 160 units of benefit, we're going to take that from you and we're going to give it to the producers instead.

The second part, in addition to the access fee, the producer has to decide the price per unit when the consumer is consuming the bundle. And they also have to decide how many songs to bundle. The way the producer makes this decision is by setting the price per unit equal to the marginal cost. So they're basically going to say, I'm going to throw in as many songs into this bundle, get you using as many of the songs as possible, or listening to the songs as possible. But I'm going to only throw in as many songs until it starts costing me more than it could potentially bring me by capturing your consumer surplus. So we're going to set the price per unit equal to the marginal cost equal to 0 in this problem.

Now looking at our graph, I've tossed up the demand curves for the student and the non-students. And what's going to happen in this problem is in the equilibrium case, our supply curve is actually straight along the x-axis because our marginal cost is equal to 0. So if this was a competitive equilibrium, the producer's surplus would be equal to 0.

But what we're going to do in this case, or what the producers are going to do in this case, is they're going to say I can bundle for the students 160 songs, so that the quantity produced is here. I'm going to charge you the marginal cost equal to 0 for each song you listen to. But then I'm going to come back and I'm going to make you pay all of your consumer surplus each year to download those 160 songs.

So again, I went to the marginal cost, which is right at 0. Then I set the price where it intersects with the demand curve. And then again, I said that we're going to charge an access fee equal to that area. The same thing is going to happen to the non-students. In addition to charging the consumer surplus for A, we're going to make a larger bundle with 200 songs, so that we can capture this additional consumer surplus out here. And I'm going to break this area up into two pieces called B and C. And you'll see why I'm doing that in just a second.

So for part A, how big are we going to make the bundles for the students and the non-students? The bundle for the students is going to be equal to 160. And the bundle for the non-students is going to be equal to 200.

Now for part B, we know that we're going to charge a price per unit equal to 0. We just need to calculate what the access fee for each of these groups is going to be. And so the access fee for the group of students who have a song bundle of 160 is just going to be the area of triangle A. So their fee is going to be consumer surplus of students, which is equal to A, which is going to be equal to  $\frac{1}{2}$  times 160 squared. So we're going to charge the consumers a fee that's

going to be equal to 12,800.

And a common mistake on this problem would be to just have this answer and put dollars here. But if you look back on the problem it said that all prices are given in cents. So we know that the fee for the students is going to be \$128. And now we know that the fee for the non-students is going to be higher because their consumer surplus is greater when they have a bundle of 200. So the fee for the non-students is just going to be the consumer surplus of the non-students. And that's going to be the area of A plus B plus C. Which again, that's going to just be equal to  $1/2$  times 200 squared. So in this case, our fee for the non-students is going to be equal to \$200. So when the monopolist has a way of telling who's a student and who's a non-student, then they can charge the non-students a higher price because their demand curve is further up and they're willing to pay for more songs within a bundle.

Part B is going to ask us, or part C, moving on to part C. It's going to ask us, what's the maximum price that students are willing to pay for this bundle? So for the 160 song bundle, we're asked, what's the maximum price students are willing to pay?

Well, if we charge them a price higher than their consumer surplus of \$128, then they're just going to drop out of the market. They're going to say, I'm not going to even buy this bundle. You're taking away all of the benefit and then an extra one penny. It's worth it for me to just go someplace else and take my money elsewhere. So the answer to part C, the maximum price you can charge a consumer in a student for the 160 song bundle is just the \$128 that we calculated before.

Part D brings up an interesting implication, however, for the non-students who can look over and see the maximum price that we're charging the students for their bundle. Part D says, what is the gross consumer surplus that non-students enjoy if they consume 160 songs per year at the price from C? What is the net consumer surplus?

So gross consumer surplus is just going to be the situation that we had before. In this case, we're looking at this point here. We're dealing with the non-students. They're going to now be consuming the 160 song bundle. The consumer surplus for this group of students is going to be both A and B. So working with part C, we know that the gross consumer surplus for the non-students when they're consuming 160 songs is going to be the area of A plus B. And we know that the net consumer surplus, they're going to be charged a fee equal to A. So the net consumer surplus is just going to be what we calculate for the gross consumer surplus minus

the fee the area of A, 128.

So even though the non-students will get no consumer surplus in net if they go with the 200 song bundle, maybe if they go with the 160 song bundle, they'll be left over with some consumer surplus.

Now to calculate the area of A plus B, all we're going to do is we're going to say that we know the area of A plus B plus C. Now we're just going to subtract out the area of C.

And if we were to go back to our demand curve for the non-students, plugging in the quantity of 160 into that demand curve, we would find that it intersects this axis at 40. So the area of A plus B plus C is going to be equal to 200. And then the area of C is going to be equal to  $\frac{1}{2}$  times 40 squared. And so that means that the gross consumer surplus for a non-student is going to be equal to \$192. So that's the area of A plus B in our diagram.

Now to find the net consumer surplus, we're just going to subtract out 128. And we're going to find that the net consumer surplus for a non-student who's consuming 160 songs is going to be equal to \$64.

So if the monopolist was still to price the same prices that they had before, if they were to make the 160 song bundle \$128 and the 200 song bundle \$200, in that case, all of the non-students would look at their two choices between the bundles and they'd say, even though this non-student bundle is targeted towards me, I'm better off in net consumer surplus if I go with the bundle that's targeted toward students.

But now the monopolists aren't going to be stupid in this situation. They're going to go back and they're going to think, well, if you're going to go and buy the non-student bundle, then what I have to do is I have to make sure-- if you're going to go back and buy the student bundle, I have to make sure that the non-student bundle gives you a higher net consumer surplus, so that you're still willing to buy the non-student bundle with 200 songs.

Part B asks us, what is the maximum price Napster can charge for 200 songs per year if it offers 160 songs per year at the highest price that students are willing to pay?

And so simply put, what they can charge for the 200 song bundle is they can charge for an area of A and C. So we're going to subtract out. We have to leave the non-students with at least enough consumer surplus for the area of B.

We know from our previous problems, we know that the area of A is just going to be equal to  $\frac{1}{2}$  times 160 squared plus  $\frac{1}{2}$  times 40 squared. This is the area of A and this is the area of C. So we can find that the maximum fee is going to be \$136.

The other way of calculating this is to just say, well, I know that the non-students are starting off with 200 hours of consumer surplus. I know that I need to leave them with at least \$64 of consumer surplus. So you can just think intuitively. You know the maximum price is going to be \$136.

And so what we're really going to look at in this situation where they can't discriminate between the students and the non-students, this is the price that they're going to be able to charge for the 200 song bundle. For the 160 song bundle, they can charge the maximum price the students are willing to pay.

And if we were in a scenario where there were only two customers in this market, one non-student and one student, then we could calculate the overall profits for the monopolist. And so in this scenario, the overall profits are going to be \$264.

The last part of this problem is going to give you another scenario that the monopolist might consider. The last part of these problems is going to ask you to look at, what happens if instead of designing a student bundle that has 160 songs, what happens if we design a student bundle that instead is only going to have 140 songs?

And again, when you're designing this bundle, you're going to have to plan the prices so that only students will want the 140 song bundle and only non-students are going to want the 200 song bundle. And the process is going to be the same.

So we're going to stop here on this problem. And just to summarize what we looked at here, we looked at the consequences and the implications of having a price-discriminating monopolist. When you can actually discriminate, you can capture all the consumer surplus and you can charge different prices to different groups in the market. But when you can't discriminate, what you have to do is you have to make packages, bundle them and price them, so that the consumers will discriminate themselves by picking the bundle that gives them the highest net consumer surplus.

I hope you found this part of the problem helpful. And go ahead and continue through with part I and check your answers against the solution.