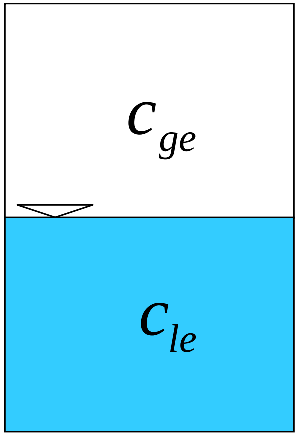
8 Surface Processes

- Mass Exchange
  - Volatilization
  - Reaeration
- Momentum Transfer
  - Oil spills
- Surface Heat Transfer
  - Lake temperature models

Air water exchange

Equilibrium: Henry's Law

$$H = \frac{c_{ge}}{c_{le}}$$



Typical units for [H]:

atm-m^3/mol (K_H) or dimensionless (K_H')

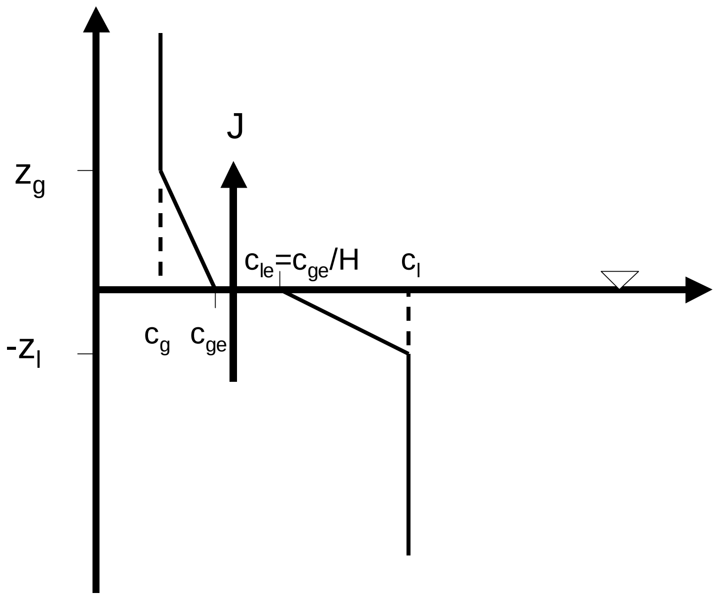
For air $K_H' \sim 42\ K_H$

Two-film theory

Ex: liquid side

$$J_l = k_l (c_l - c_{le})$$

$$J_l = D_l \frac{(c_l - c_{le})}{z_l}$$



$$k_l (c_l - c_{le}) = k_g (c_{ge} - c_e) = k(c_l - c_g / H)$$

$$H = \frac{c_{ge}}{c_{le}}$$

3 eqns, 3 unknows (c_{le}, c_{ge}, k)

Two-film theory

$$\frac{1}{k} = \frac{1}{k_l} + \frac{1}{Hk_g}$$

1 2

$$c_{ge} - c_g = \frac{k_l (Hc_l - c_g)}{Hk_g + k_l}$$

$$c_l - c_{le} = \frac{k_g (Hc_l - c_g)}{Hk_g + k_l}$$

Resistances in series:

$k_l << Hk_g$ => 1 dominates (liquid side controlled)

$k_l >> Hk_g$ => 2 dominates (gas side controlled)

Medium with lower equilibrium concentration controls

Typical values for air and water

"Typical" values for air (as gas)

$D_g \sim 0.1\ cm^2/s$, $0.1 < z_g < 1\ cm$

$\Rightarrow k_g = D_g/z_g = 0.1$ to $1\ cm/s$

"Typical" values for water (as liquid)

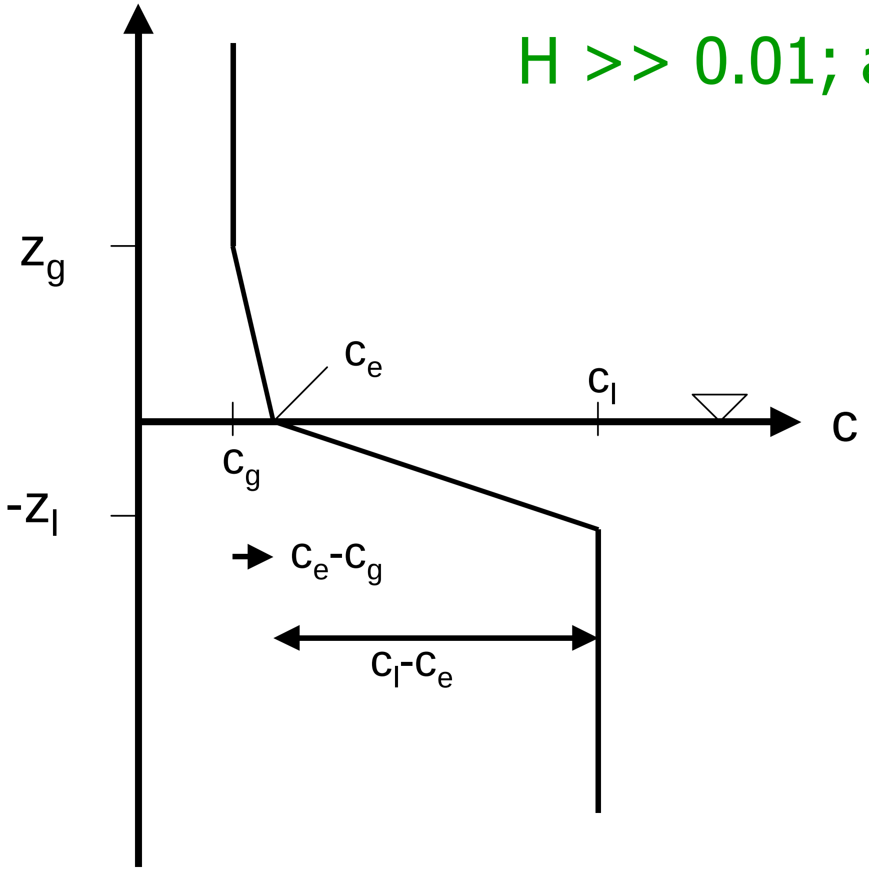
$D_l \sim 2x10^{-5}\ cm^2/s$, $0.002 < z_l < 0.02\ cm$

$\Rightarrow k_l = D_l/z_l = 10^{-3}$ to $10^{-2}\ cm/s$

$K_g \sim 100\ k_l$ so if $H >> 0.01$ then water side controlled (think DO); if $H << 0.01$ then air side controlled (think evaporation)

Example of liquid side control

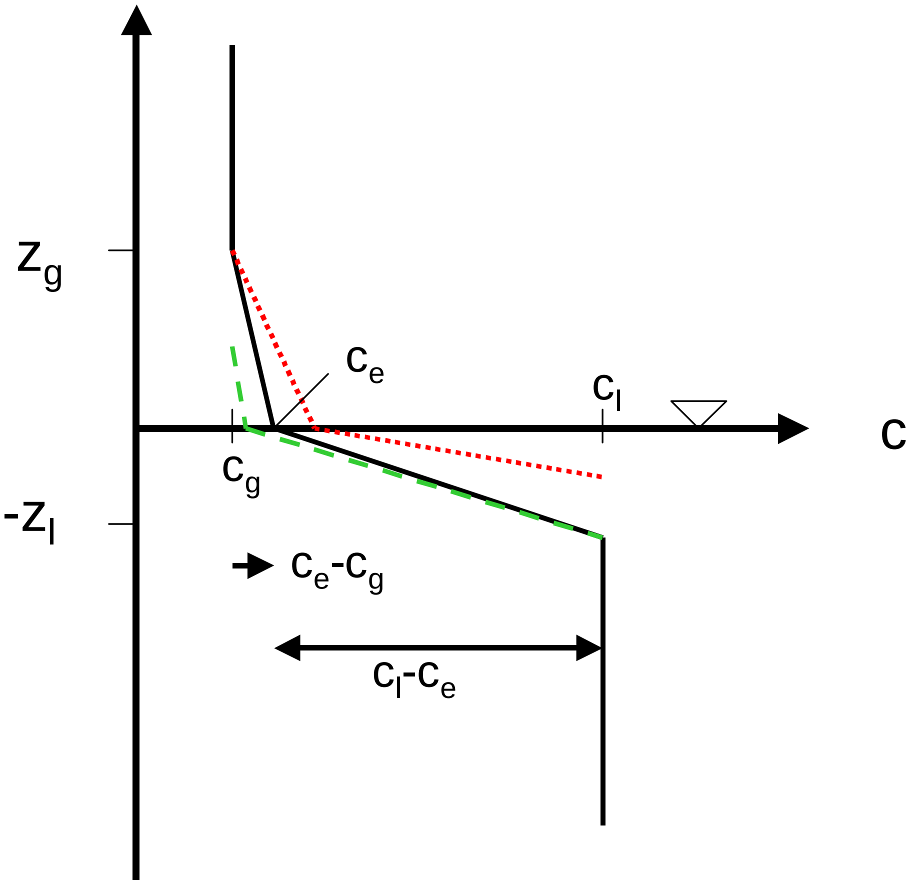
$H >> 0.01$; assume $H \sim 1 \Rightarrow c_{ge} = c_{le} = c_e$



$$c_e - c_g = \frac{k_l(Hc_l - c_g)}{Hk_g + k_l} \cong (c_l - c_g)\frac{k_l}{k_g}$$

$$c_l - c_e = \frac{k_g(Hc_l - c_g)}{Hk_g + k_l} \cong (c_l - c_g)$$

Liquid side control, cont'd



If we double k_l (halve z_l), (c_e-c_g) doubles, both gradients ~ double => twice the mass flux; red line

If we double k_g (halve z_g), (c_e-c_g) is halved, both gradients ~ const => similar mass flux; green line

Therefore mass flux controlled by liquid side

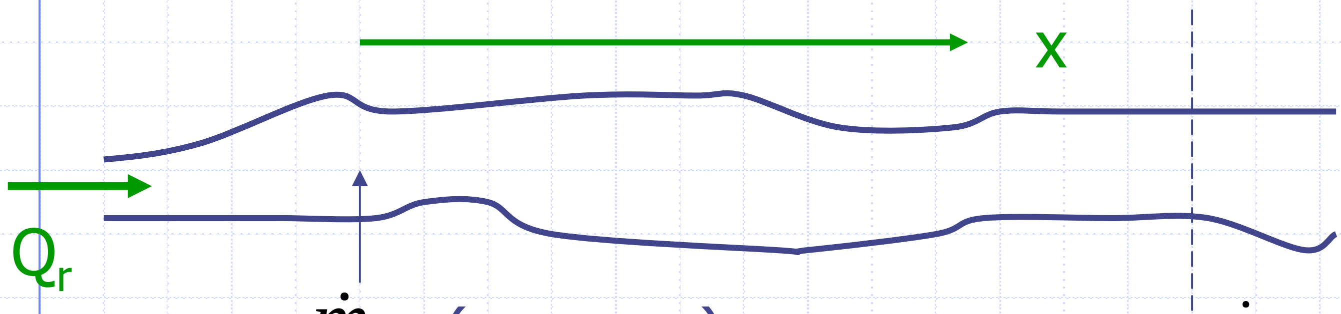
Surface Renewal Theory

- Described previously for stream-reaeration formulae (Chapter 7)
- z_l (or z_w or δ) not stagnant, but time-dependent $\sim [Dt]^{1/2}$, where t is reciprocal of a renewal rate, depending on bottom generated turbulence.
- Thus k_l (hence k) $= D_l/z_l \sim D^{1/2}$

Measurement of gas exchange

- Gas-evasion experiment: introduce chemically conservative gas (e.g., CO_2, propane, radon) at $c >$ saturation, and watch c decline with distance due to volatilization
- In open water bodies (or rivers where you don't know flow rate) introduce a second, non-volatile tracer such as salt.
- Sometimes use tracer of opportunity

Application to rivers



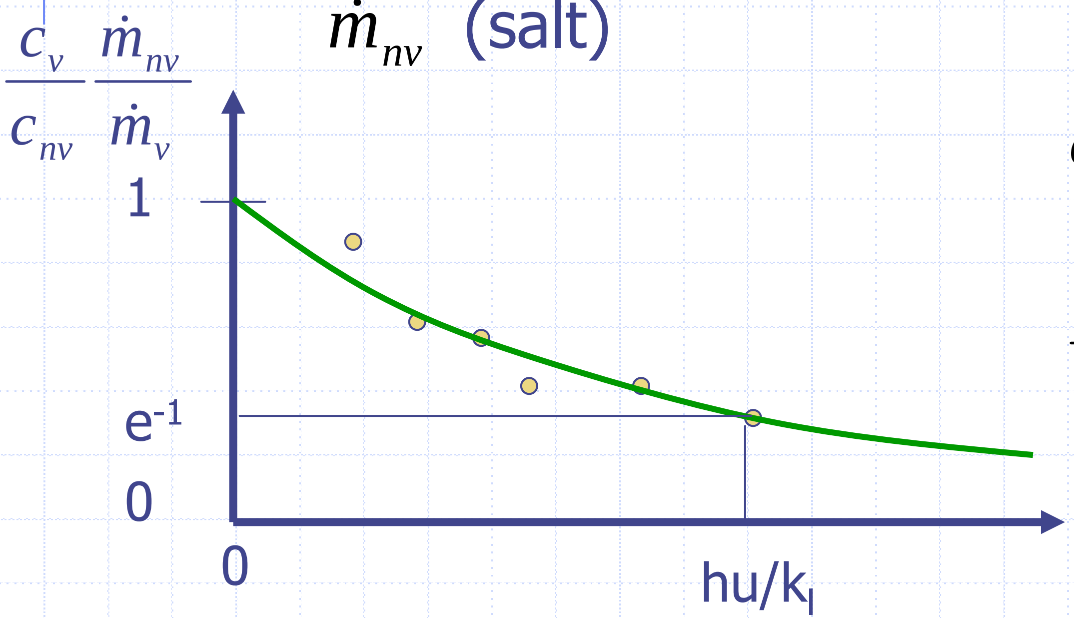
$\dot{m}_v$ (propane)

$\dot{m}_{nv}$ (salt)

$$c_{nv} = \frac{\dot{m}_{nv}}{Q_r}$$

$$c_v = \frac{\dot{m}_v}{Q_r} e^{-k_l x / hu}$$

$$\frac{c_v}{c_{nv}} = \frac{\dot{m}_v}{\dot{m}_{nv}} e^{-k_l x / hu}$$



(stream reaeration coefficient $K_a = k_l/h$)

Gasses other than oxygen

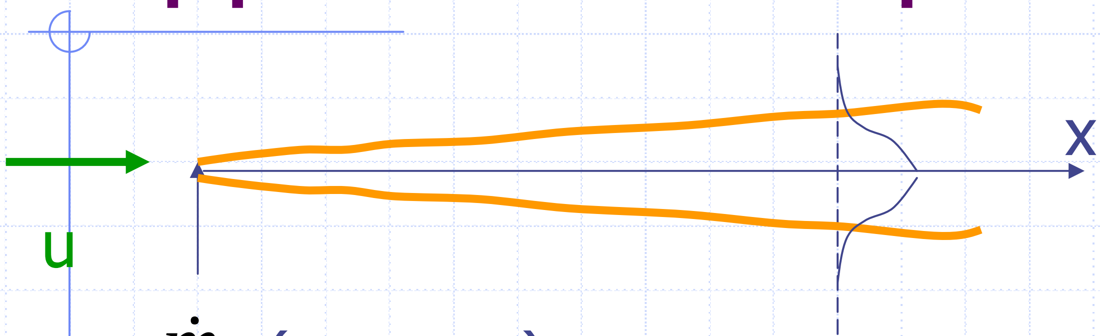
- $K_a \sim D$ (stagnant film), $D^{1/2}$ (surface renewal), $D^{2/3}$ (split the difference)
- From Chapter 1, $Sc = \nu/D \sim MW^b$ ($b \sim$ 0.35 to 0.4)
- $K_a/K \sim (D_{O2}/D)^{2/3} \sim (32/MW)^{-1/4}$
- Example: Propane C_3H_8, MW = 44
  - $K_a/K = (32/44)^{-1/4} = 1.08$
  - Calibrations actually shows $K_a/K \sim 1.39$

How far downstream must one go?

- O'Connor-Dobbins at 20°C:

  $K_a = 3.9u^{0.5}/h^{1.5}$
- $u = 0.3$ m/s, $h = 1$ m, $K_a = 2.1\ d^{-1}$
- $x \sim u/K_a = (0.3\ m/s)(86400 s/d)/(2.1 d^{-1}) \sim 12$ km

Application to open waters



h = water depth or thermocline depth

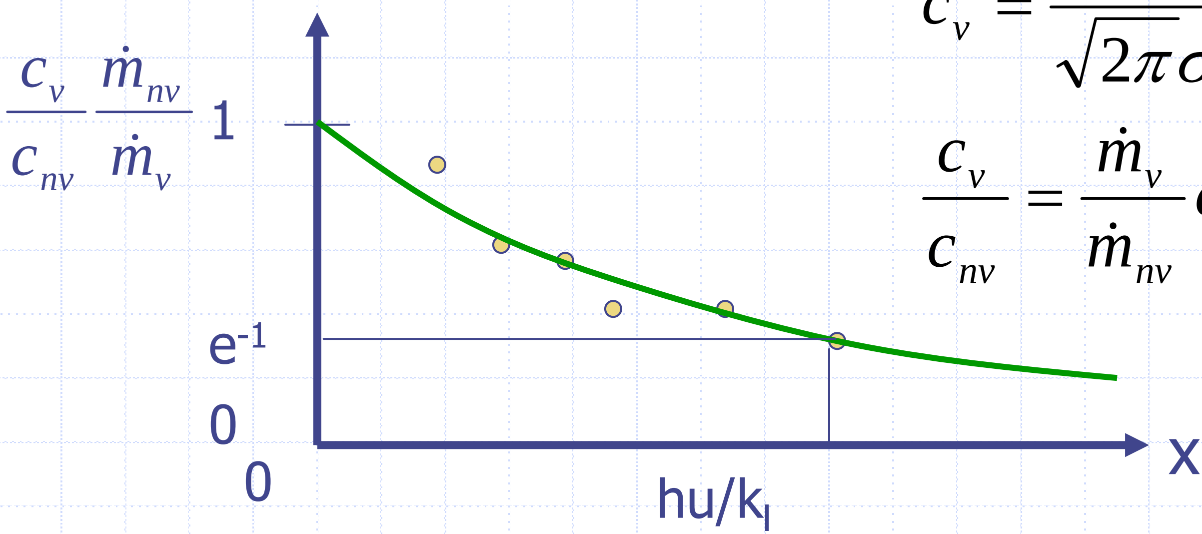
$\dot{m}_v$ (propane)

$\dot{m}_{nv}$ (salt)

$$c_{nv} = \frac{\dot{m}_{nv}}{\sqrt{2\pi}\sigma u h} e^{-y^2/2\sigma^2}$$

$$c_v = \frac{\dot{m}_v}{\sqrt{2\pi}\sigma u h} e^{-y^2/2\sigma^2} e^{-k_l x/hu}$$

$$\frac{c_v}{c_{nv}} = \frac{\dot{m}_v}{\dot{m}_{nv}} e^{-k_l x/hu}$$



Mass transfer in lakes and oceans

- Most contaminants of concern are water side controlled (e.g., DO, VOC)
- In rivers, source of turbulence is bottom roughness
- In deep water bodies (lakes, oceans) it is wind stress => u_{w*} (water-side friction velocity) which affects z_l
- Contaminants that are air side controlled also affected by wind (through z_g)

k_l vs u_{w*}

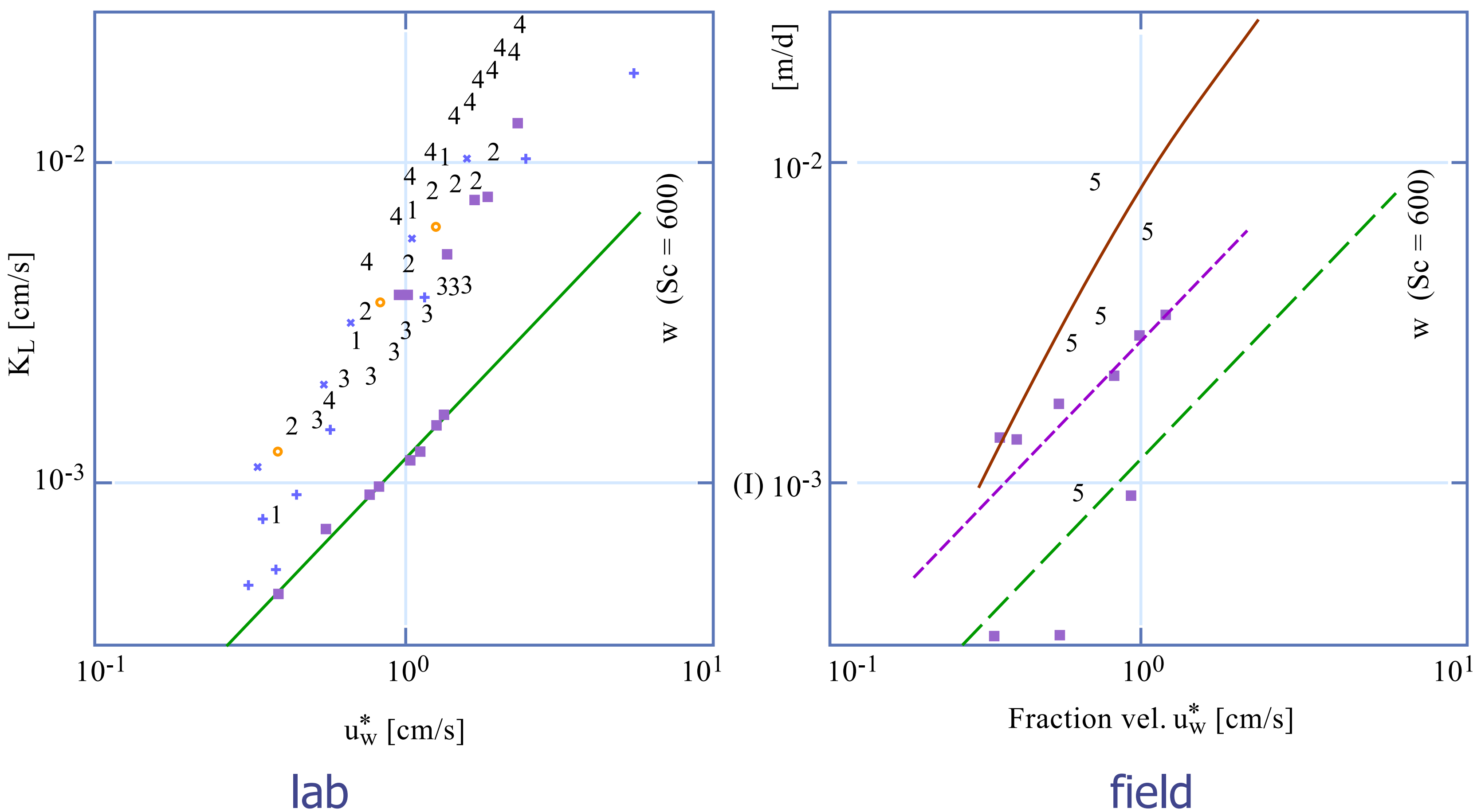
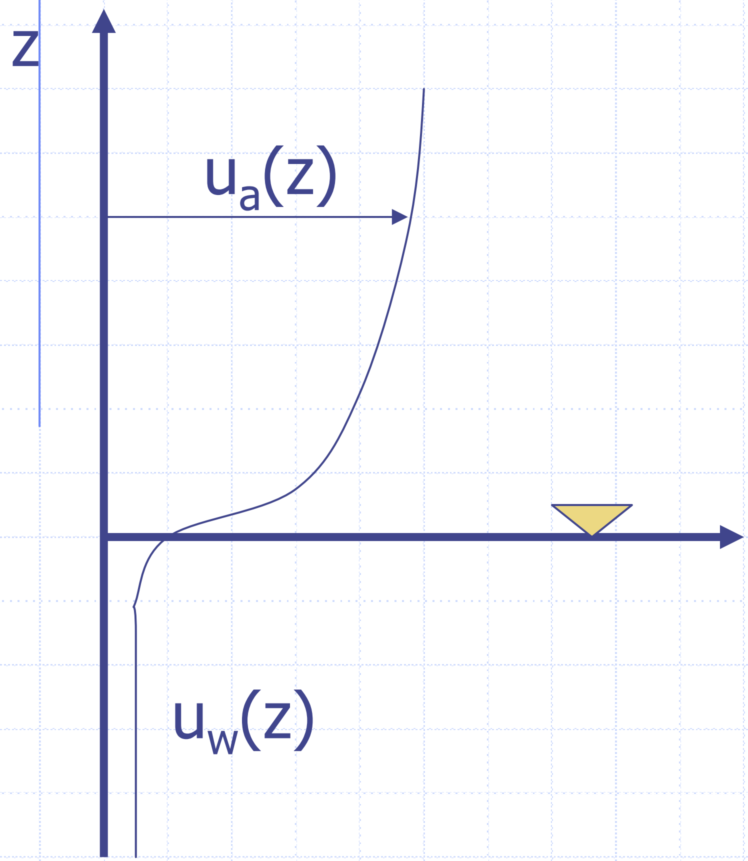


Figure by MIT OCW.

u_{w*} because transfer is water side controlled and u_{w*} is indicator of turbulence; yet u_{w*} not easily measured

Wind Stress



$$\tau = C_{10}\rho_a u_{10}^{\;2} = \rho_w u_{w*}^{\;2}$$

$$u_{w*} = \sqrt{\frac{C_{10}\rho_a}{\rho_w}}u_{10}$$

u_{10} = 10 m wind speed;

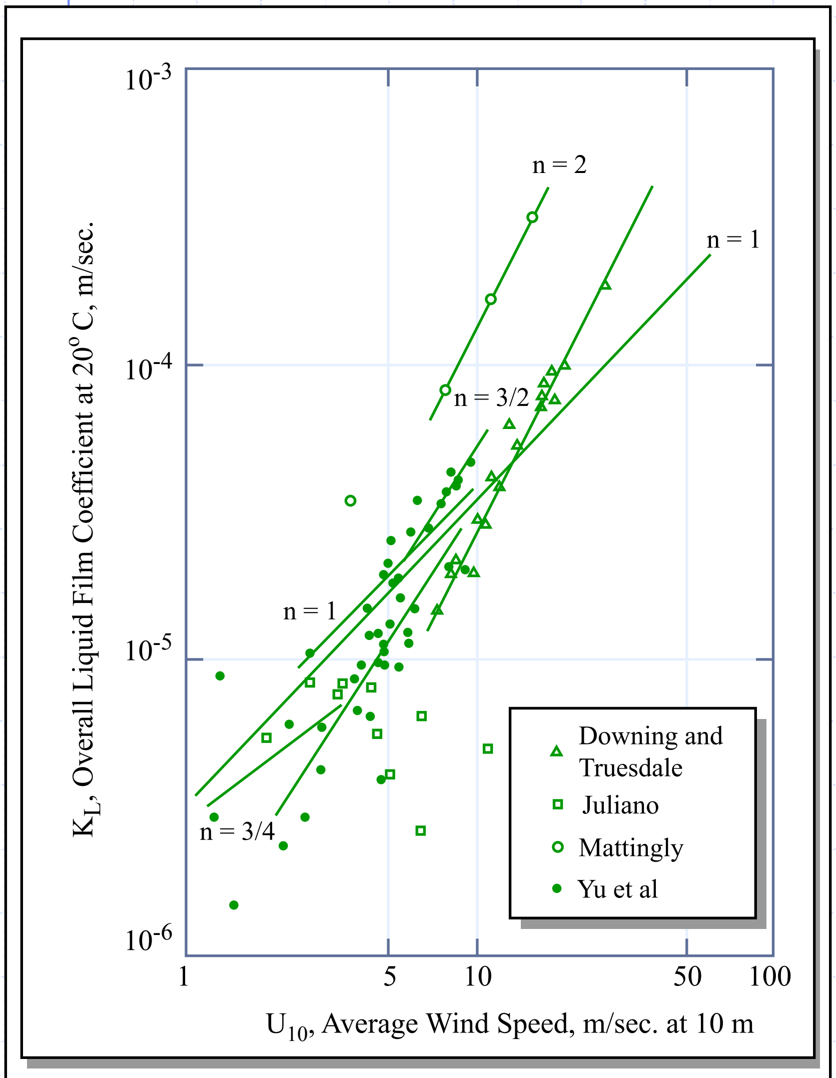
C_{10} = drag coef.

$C_{10} = (0.8+0.065u_{10}) \times 10^{-3}$

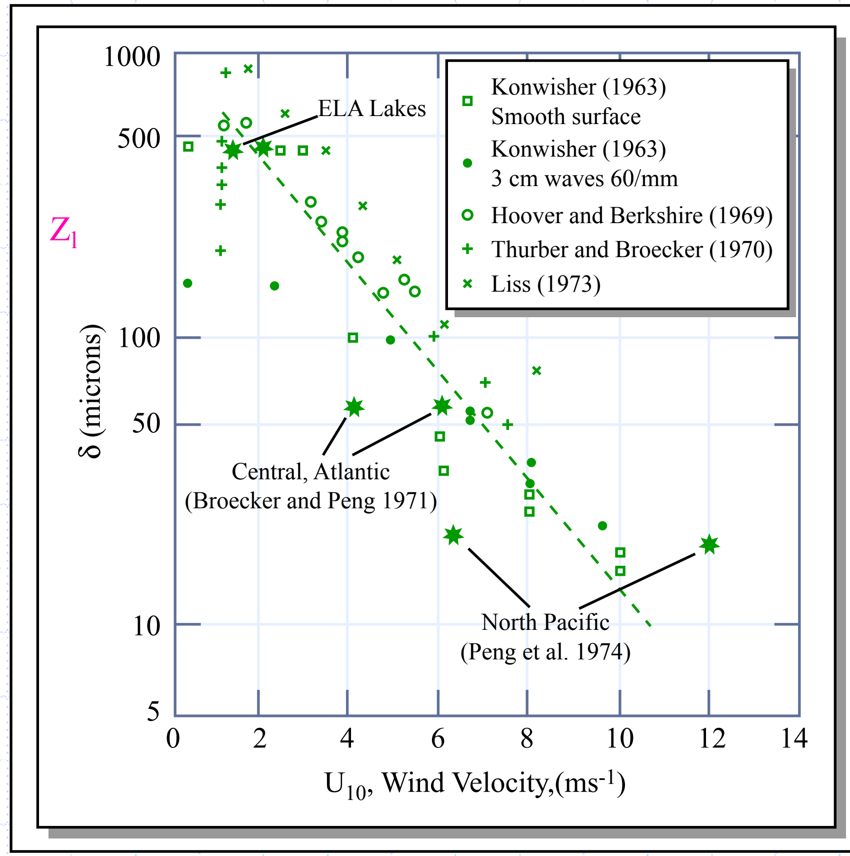
[u_{10} > 1 m/s; Wu, 1980]

u_{10} -> C_{10} -> u_{w*} -> k_l

k_l (or z_l) vs u_{10}



Yu and Hamrick (1984)



Figures by MIT OCW.

Emerson (1075)

Example film coefficients

$$k_l = 0.0004 + 0.00004{u_{10}}^2$$

$$k_g = 0.3 + 0.2u_{10}$$

k_l and k_g in cm/s; u_{10} in m/s [Schwarzenbach et al, 1993]

Note that both depend on u_{10}

Examples

Above eqns:

$u_{10} = 5$ m/s => $k_l = 1.4 \times 10^{-3}$ cm/s (green dot);

$k_g = 1.3$ cm/s

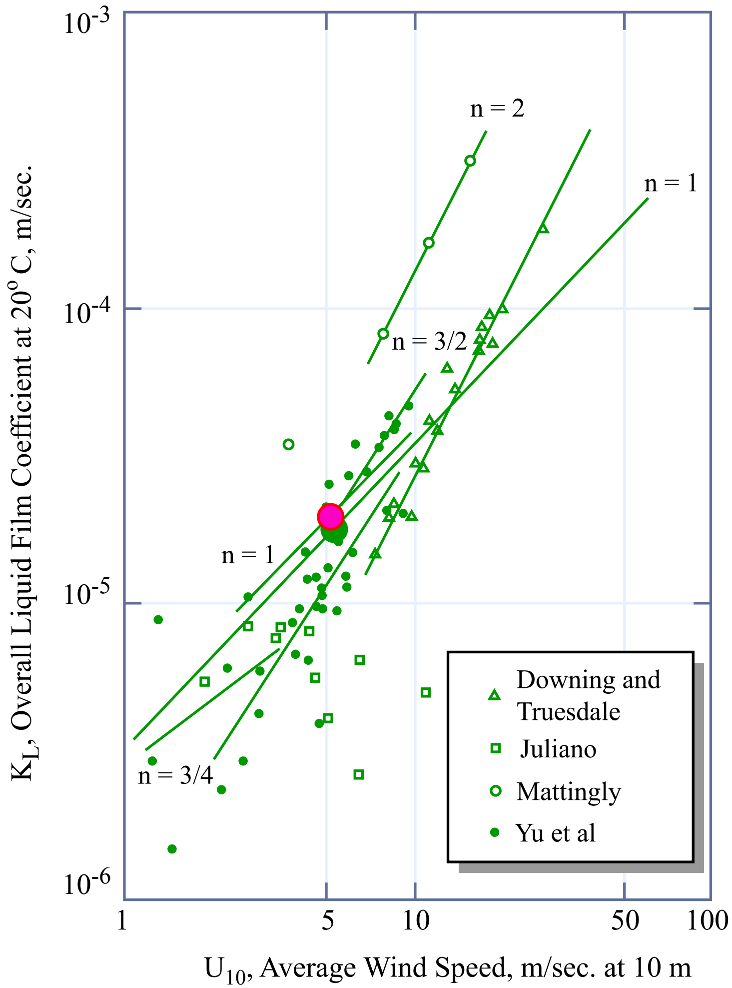
Figure 8.8:

$z_l = \delta = 120\ \mu m = 1.2 \times 10^{-2}$ cm.

For DO, $D = 2 \times 10^{-5}\ cm^2/s$

$k_l = D/z_l = 2 \times 10^{-5} / 1.2 \times 10^{-3} = 1.7 \times 10^{-3}$ cm/s (red dot)

k_l (or z_l) vs u_{10}



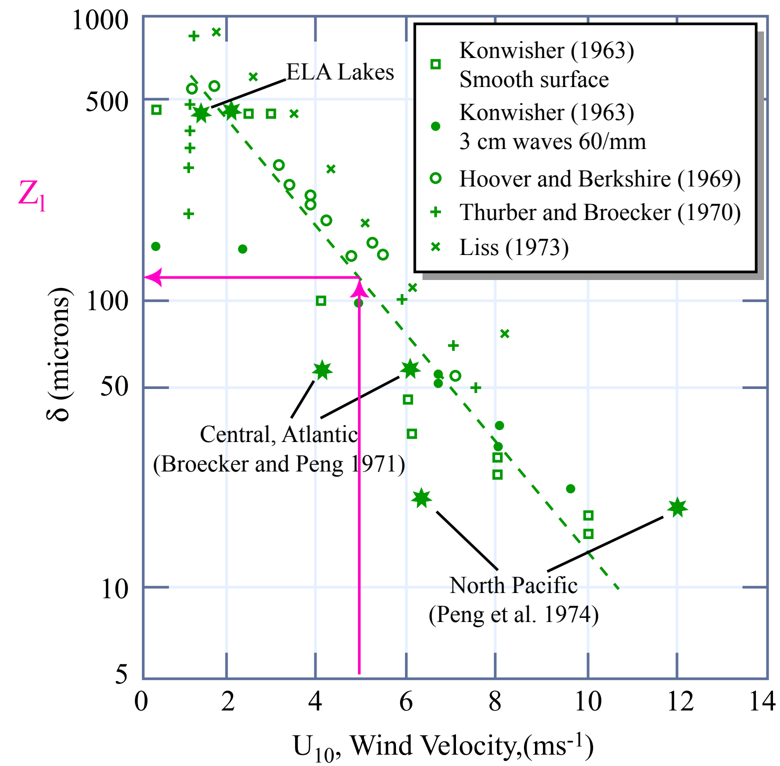


Figure by MIT OCW.

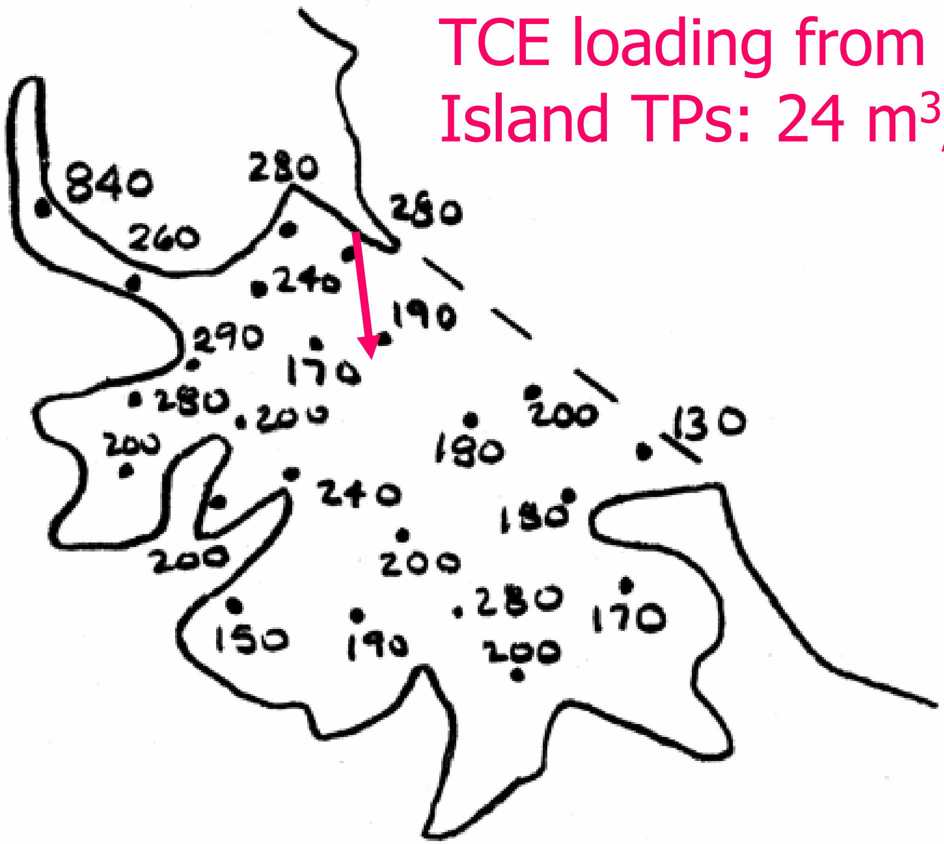
Yu and Hamrick (1984)

Emerson (1075)

Volatile Halogenated Organic Compound (VHOC) Experiment

- CH_3Cl_3 and other one carbon VOCs (THMs) and two carbon VOCs (solvents) discharged with waste water.
- Used to
  - compute volatilization (assuming known residence time) or
  - compute residence time (with known volatilization)

TCE data in Boston Harbor



TCE loading from Deer & Nut Island TPs: 24 m^3/s at 11 $\mu g/L$

Ave harbor TCE concentration

241 ng/L (all pts)
214 ng/L (excl. 840)

Harbor volume = 6×10^8 m2

Kossik, Gschwend & Adams, 1987

TCE Experiment, cont'd

Nominal residence time (w/o volatilization; excluding presumed outlier)

$$\tau^* = \frac{\bar{c}V}{Q_o c_o} = \frac{(214x10^{-9}\,kg/m^3)(6x10^8\,m^3)}{(24m^3/s)(86400s/d)(11x10^{-6}\,kg/m^3)} = 5.6d$$

TCE Experiment, cont'd

With volatilization

$$V\frac{dc}{dt} = -kAc - Q_f$$

$$\frac{dc}{dt} = -\underbrace{\left(\frac{k\ A}{V} + \frac{Q}{V}\right)}_{\kappa^* = 1/\tau^*}c$$

k = piston velocity ~ k_l (water side control)

κ^* = bulk removal rate (t^{-1})

V/Q = τ = hydraulic res time

kA/V = k/h

For CH_3Cl_3 H = 1.13 (dimensionless) >> 1 => ws control

D = 1.0×10^{-5} cm^2/s

TCE Experiment, cont'd

From Figure 8.8 and u_{10} = 5 m/s, $\delta = 1.2\text{x}10^{-2}$ cm

$k = D/\delta = (1.0\text{x}10^{-5})/(1.2\text{x}10^{-2}) = 0.00083$ cm/s = 3 cm/hr = 0.72 m/d

$1/t^* = 1/t + k/h$

$1/t = 1/t^* - k/h = (1/5.6\text{d}) – (0.72 \text{ m/d})/6\text{m}$

$= 0.18 – 0.12 = 0.06\text{d}^{-1} \Rightarrow \tau = 17$ d

Estimated τ is too high; reason is likely extraneous or under-accounted sources of CH_3Cl_3

Momentum Exchange

- Chapters 2, 3 discussed surface shear stress for eddy diffusivity and hydrodynamic modeling
- Previous section discussed stress as source of turbulence governing mass exchange
- Also of interest in transporting floating material, specifically spilled hydrocarbons

Oil Spills

- Composition
- Fate
- Transport (spreading, advection)
- Clean-up

Marine Sources (10^3 MT/yr)

	N. America	Global
Natural Seeps	160	600
Petroleum Extraction	3	38
Petroleum Transport	9	150
Petroleum Consumption	84	480
Total	260	1300

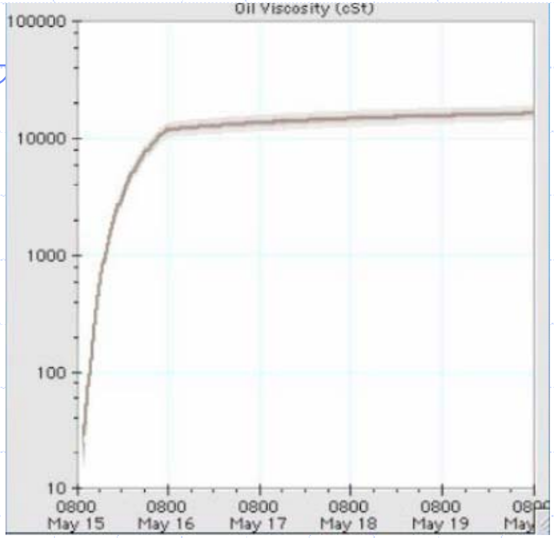
About half is anthropogenic

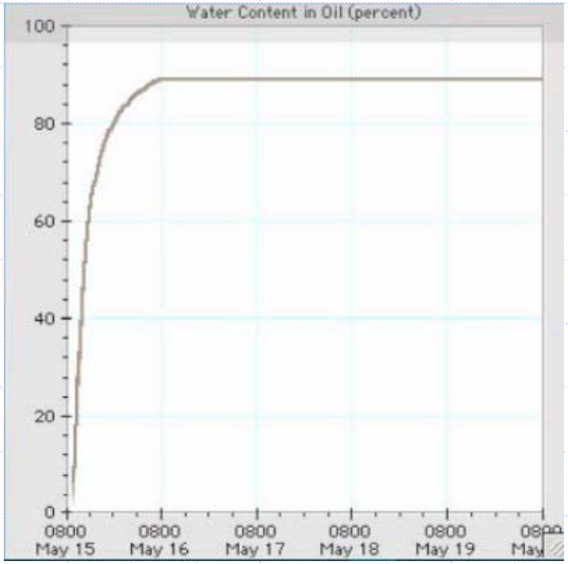
(Oil in the Sea III, NRC, 2003)

Composition

- Crude and Refined Oils
- Always multiple constituents
- Characterized by Boiling Point (or distillation cut)

Fate





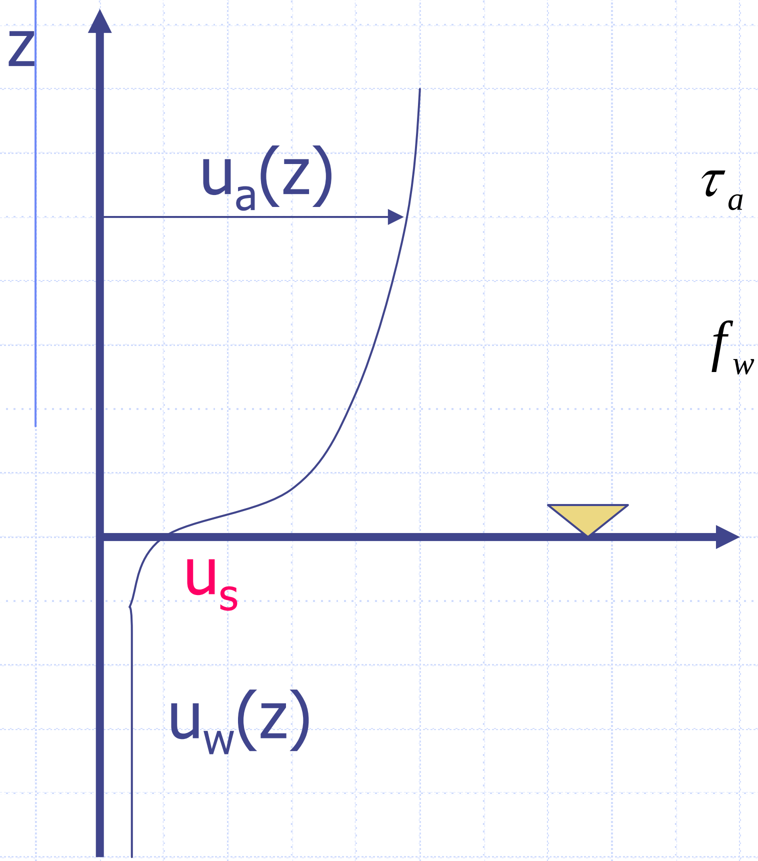
- Volatilization (lighter fractions)
- Emulsification (depending on oil)
- Natural dispersion (if enough energy)
- Biodegradation
- Dissolution
- Photo-oxidation
- Sediment particle interaction

Output from NOAA's ADIOIS model; independent of transport

Transport Models

- Spreading and Advection
- Pre-planning (evaluate risk)
- Real-time (assist clean-up; needs to be quick and dirty)
- Hind-cast (who is responsible, damage assessment)

Simple advection model



$$\tau_a = \frac{f_a}{2}\rho_a u_a^2 = \frac{f_w}{2}\rho_w(u_s - \bar{u}_w)^2 = \frac{f_w}{2}\rho_w(\Delta u_s)^2$$

$$f_w \cong f_a \quad \rightarrow \quad \frac{\Delta u_s}{u_a} = \sqrt{\frac{\rho_a}{\rho_w}} \cong 0.03$$

Surface current speed ~ 3% of wind speed. (Also explained by Stokes Drift due to surface waves)

In which direction?

Ekman Model

Linearized equations of motion; constant viscosity

$$\frac{\partial u}{\partial t} - \Omega v = E \frac{\partial^2 u}{\partial z^2}$$

$\Omega = 2\omega \sin\phi$ Coriolis parameter

$$\frac{\partial v}{\partial t} + \Omega u = E \frac{\partial^2 v}{\partial z^2}$$

$w = u + iv$ Complex velocity

$$w = 0$$

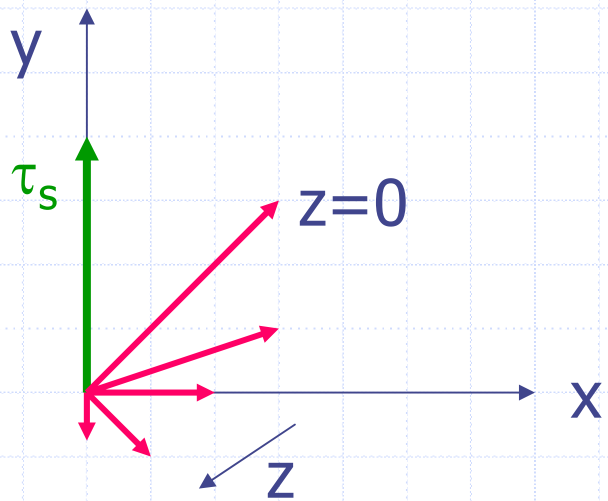
At depth (z = - oo)

$$E \frac{\partial w}{\partial z} = \frac{\tau_{sx} + i\tau_{sy}}{\rho_w}$$

At surface (z = 0)

Ekman Model, cont'd

$$w = \frac{\tau_{sy}}{\rho_w \sqrt{E\Omega}} \exp\left\{ \sqrt{\frac{\Omega}{2E}}(1+i)z + \frac{i\pi}{4} \right\}$$

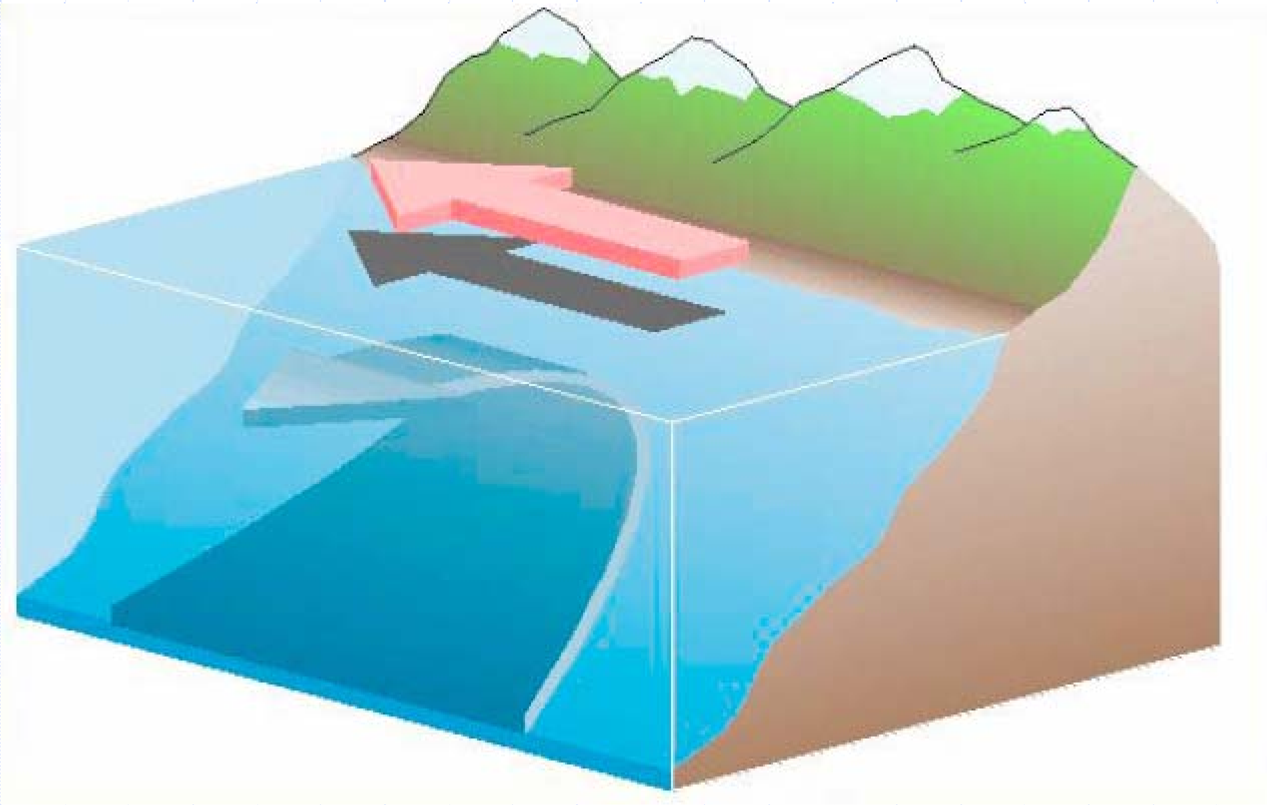


Surface drift 45° to right;

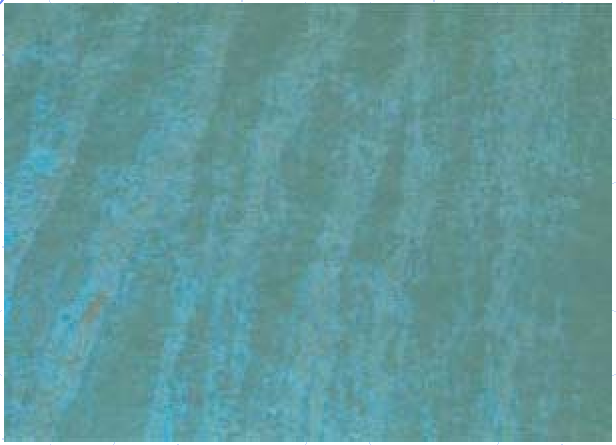
Depth average drift 90° to right

Field experiments show surface drift ~ 10° to right. Explained by variable vertical viscosity E ~ z (Madsen, 1977)

Other effects of wind: Coastal Upwelling/Downwelling



Other effects of wind: Langmuir Circulation



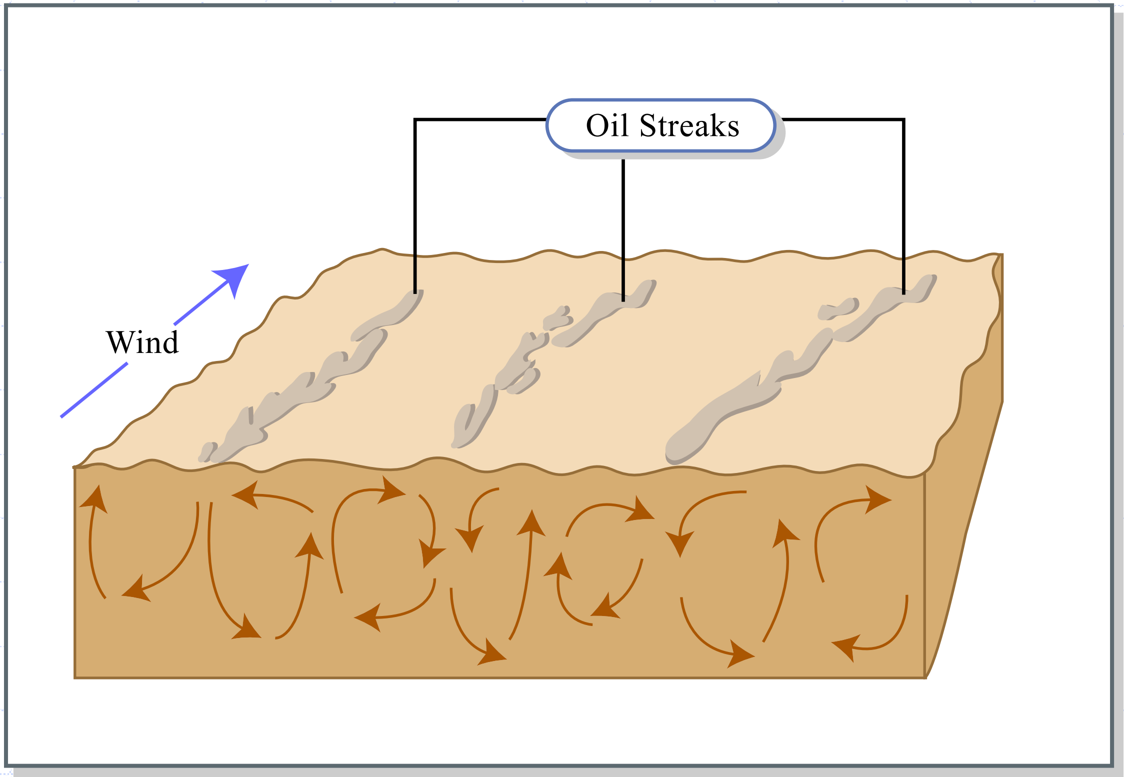
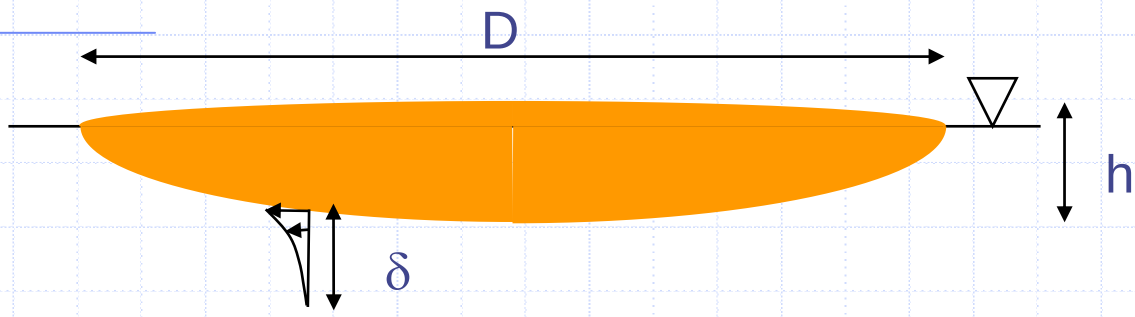


Figure by MIT OCW.

Idealized Spreading (Fay, 1969)



$$\frac{dD}{dt} \sim \sqrt{g'h} \sim \frac{1}{D}\sqrt{g'V}$$ Gravity-Inertia

$$\frac{dD}{dt} \sim \frac{g'V^2}{D^5}\sqrt{\frac{t}{\upsilon_w}}$$ Gravity-Viscous

$$\frac{dD}{dt} \sim \frac{f_r}{\rho_w D}\sqrt{\frac{t}{\upsilon_w}}$$ Surface Tension-Viscous

Idealized spreading, cont'd

Regime		
Gravity-Inertia	$D = 2k_1[g'Vt^2]^{1/4}$	$K_1=1.14$
Gravity-Viscous	$D = 2k_2[g'V^2t^{3/2}/\nu_w{}^{1/2}]^{1/6}$	$K_2=0.98$ to 1.45
Surface Tension-Viscous	$D=2k_3[f_r t^3/\rho_w 2\nu_w]^{1/4}$	$K_3=1.6$

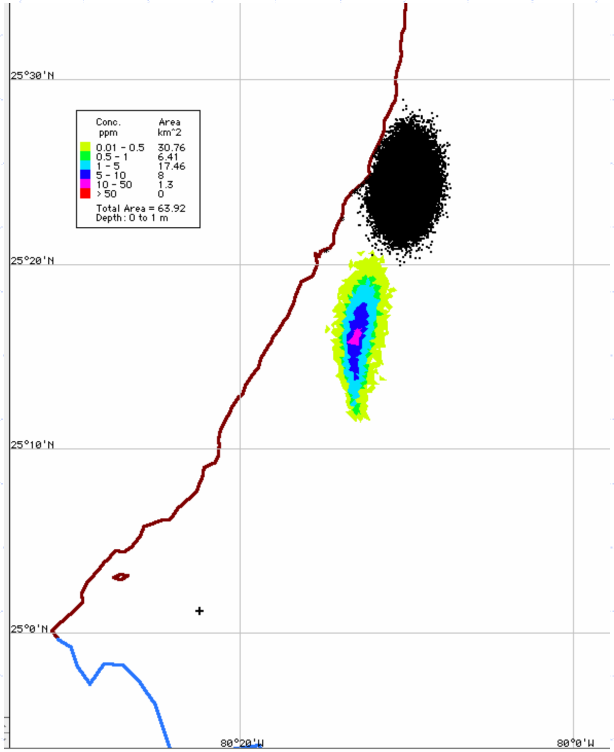
Comments

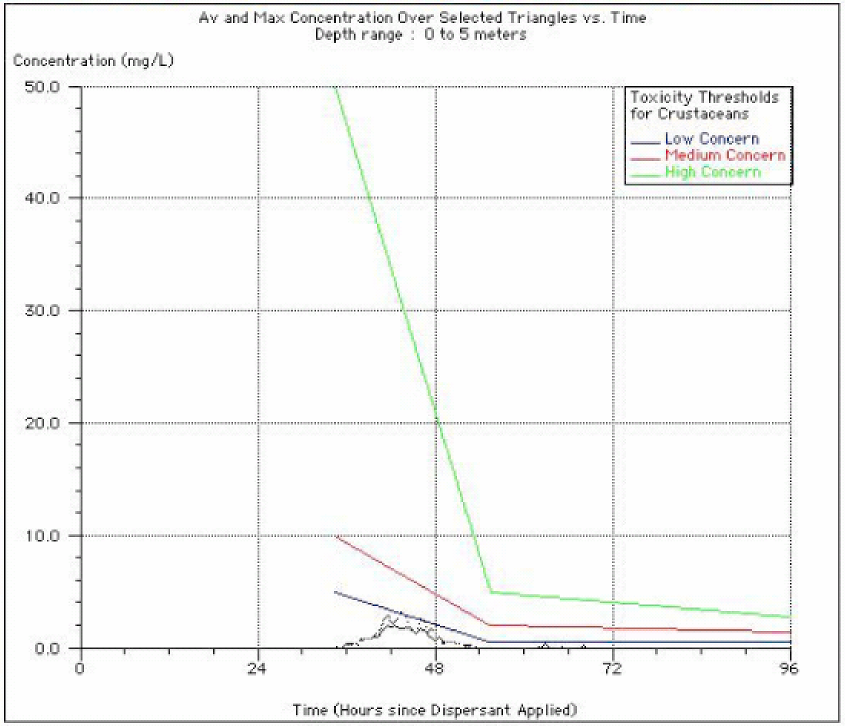
- Theory applies down to slick thickness of about 0.1 mm
- Additional spreading due to
  - Time-varying spillage
  - Wind, waves and non-uniform currents
  - Dispersion of submerged (slower moving) oil droplets
- Field experiments show oil often very non-uniform (90% of volume in 10% of area)

Oil Transport Models

- Slick advected with underlying surface current plus 3% of wind speed (~10% deflection to right)
- (3-D) models simulate transport of sub-surface dispersed oil.
- Currents can be observed or predicted (sophistication depends on application—available time)
- Fate processes often computed independently from transport

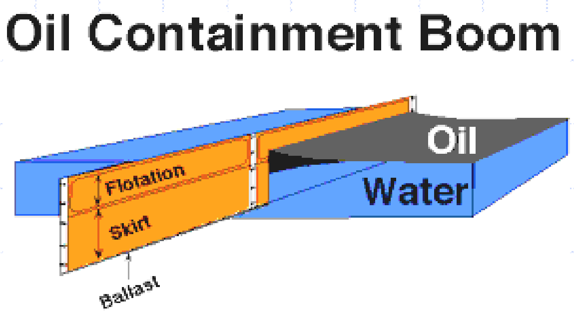
Model Simulations





NOAA's 3D GNOME; ANS Crude off Coast of Florida

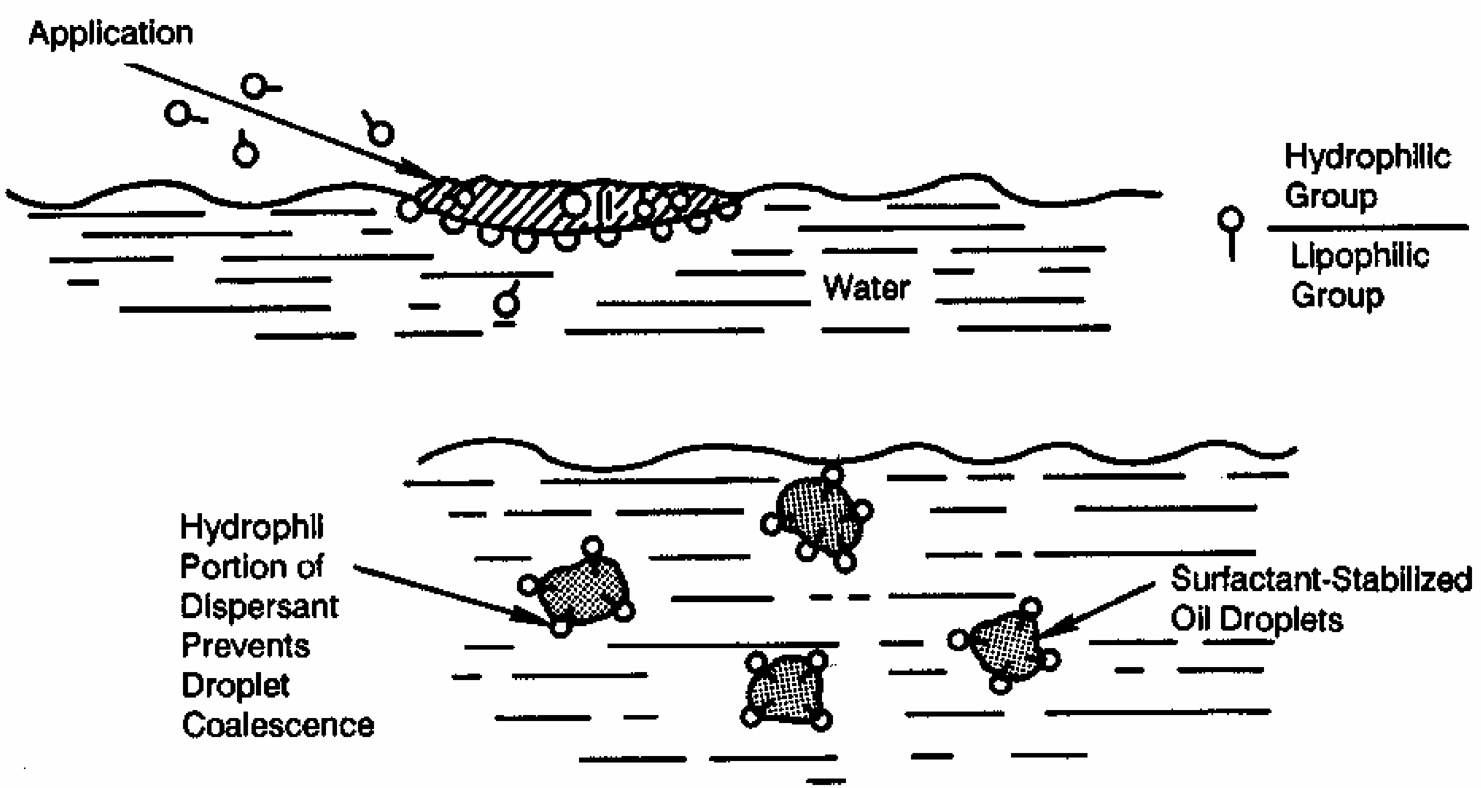
Mechanical Clean-up



Chemical Dispersion

- Surfactants that reduce interfacial tension
- Create dispersed droplets
- Subsurface/bottom impacts vs surface/shoreline
- Air (large spills) or boat application
- Window of opportunity

Chemical Dispersion, cont'd



NRC, 1989

In situ Burning

- Considered secondary option (like chemical dispersants)
- Most appropriate for offshore spills (reduced AQ impacts)

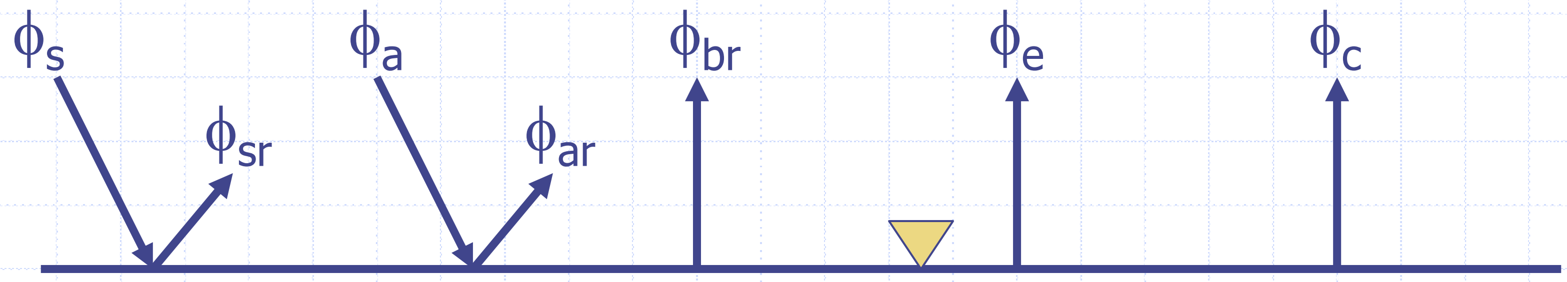
Surface Heat Transfer and Temperature Modeling

- Surface heat fluxes
- Linearized surface heat transfer
- Cooling ponds
- Natural lakes and reservoirs

Importance of Temperature

- Important WQ parameter
  - Thermal pollution
  - Species preference (fish habitat)
- Affects rate constants
  - $K = K_{20}\theta^{T-20}$
- Produces density stratification
  - $\rho = \rho(T)$
- Important tracer (e.g., E_z)

Surface Heat Transfer (W-m^{-2})



Net solar, ϕ_{sn}	60 to 300
Net atmospheric, ϕ_{an}	200 to 450
Back radiation, ϕ_r	250 to 500
Evaporation, ϕ_e	0 to 350
"Conduction", ϕ_c	-70 to 200

$$\phi_n = \phi_{sn} + \phi_{an} - \phi_{br} - \phi_e - \phi_c$$

Solar Radiation

- Short wave length (< 3μm)
- Direct plus diffuse (scattered, reflected)
- Absorbed & re-radiated (> 3μm) by clouds
- Measured by pyranometer
- Incident clear sky radiation calculated from latitude, date and time of day
- Corrections for cloud cover and reflection

Net Solar Radiation (cont’d)

ϕ_{sr}/ϕ_s (%)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	9	7	7	6	6	6	6	6	6	7	9	10

$$\phi_{sn} = \phi_s - \phi_{sr} \cong 0.94\phi_{sc}\left(1 - 0.65C^2\right)$$

C = fractional cloud cover

Depth-variation of solar radiation

- Measured with Secchi disk or *in-situ* pyranometer

$$\phi_z = (1-\beta)\phi_{sn} e^{-\eta z}$$

$$\eta = \frac{1.7}{d_D}$$

$\beta \sim 0.5$

Atmospheric Radiation

- Long wave length (> 3μm)
- Re-radiated from atmosphere
- Measured by pyrgeometer
- Incident clear sky radiation calculated from absolute air temperature, vapor pressure
- Corrections for cloud cover and reflection

Incident Radiation Formulae

$$\phi_{ac} = \varepsilon\sigma(T_a + 273)^4$$

σ = Stefan-Boltzman const ($5.7x10^{-8}$ W/m^2-$^oK^4$)

ε = emissivity (dimensionless)

$$\varepsilon = 0.92x10^{-5}(T_a + 273)^2$$ Swinbank (1963)

$$\varepsilon = \{1.0 - 0.26/\exp[7.77x10^{-5}(T_a)^2]\}$$ Itso-Jackson (1969)

$$\varepsilon = 1.24\left(\frac{e}{(T_a + 273)}\right)^{1/7}$$

e = vapor pressure, mbar

Brutsaert (1975

Net Atmospheric radiation

$$\phi_{an} = 0.97\varepsilon\sigma(T_a + 273)^4\left(1.0 + 0.17C^2\right)$$

C = fractional cloud cover

~3% reflection

Back Radiation

- Water surface is nearly a black body
  - (ε ~ 0.97)

$$\phi_{br} = 0.97\sigma(T_s + 273)^4 = 5.5x10^{-8}(T_s + 273)^4$$

Evaporative Heat Flux

- Measured
  - eddy flux (short term)
  - evaporation pans (long term)
- Computed from mass transfer formulae

$E = \rho\ f(W_z)\ (e_s - e_z)$ Dalton's Law

e_s = vapor pressure at surface

e_z = vapor pressure at elevation z

$f(W_z)$ = wind speed function = $a + bW_z$ (k_g)

Evaporative Heat Flux (cont'd)

- Mass transfer => heat transfer using latent heat of vaporization

$$L_v = (2493. - 2.26T_s) \times 10^3$$ J/Kg

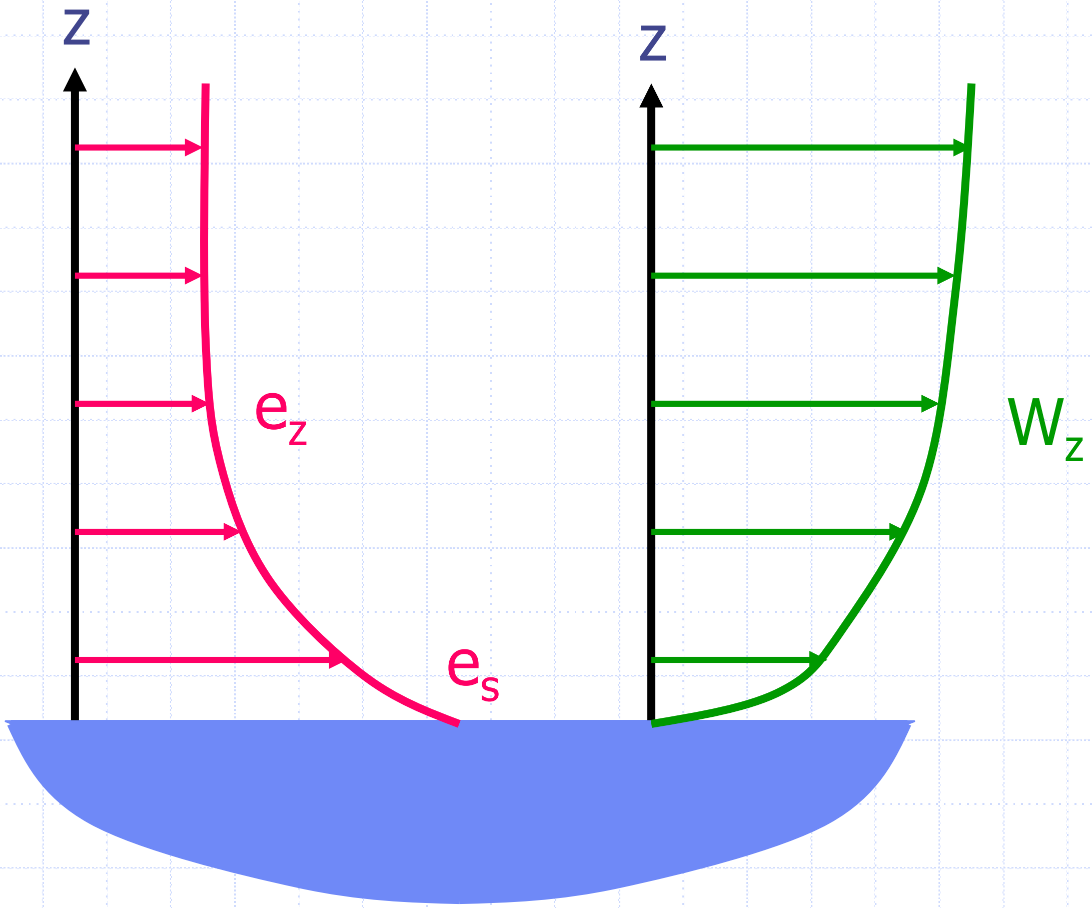
$$\phi_e = L_v E = f(W_z)(e_s - e_a)$$

$$\phi_e = 3.72W_2(e_s - e_2)$$ "Lake Hefner", Marciano and Harbeck (1954)

(W/m^2; W_2 in m/s; e_s, e_2 in mb)

$$\phi_e = 5.1A^{-0.05}W_2(e_s - e_2)$$ "Fetch-dependence" Harbeck, (1962)

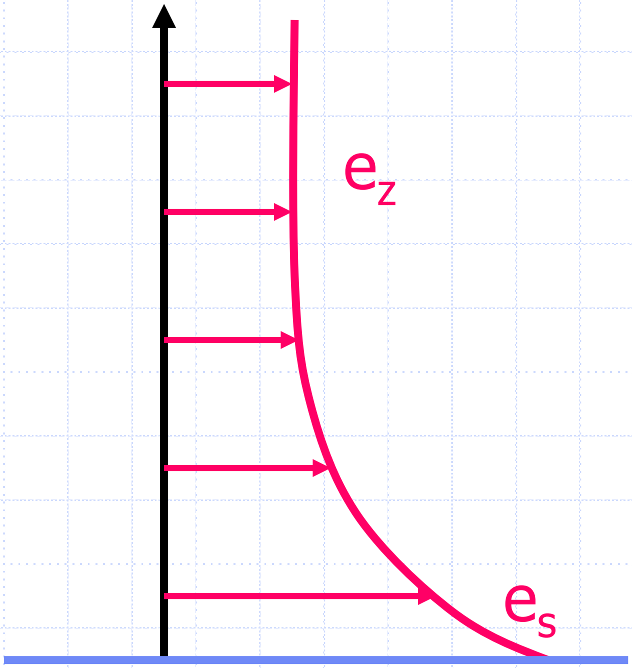
(A in ha)



e_z and W_z vary vertically (height above water) and horizontally (above water or on-shore)

Evaporation from non-natural water bodies

- e_s increases with temperature
  - Heated water bodies have increased evaporation (water vapor also lighter than air)
- e_s decreases with salinity
  - Saline bodies have decreased evaporation
- e_s decreases with pressure



$\phi_e = f(W_z)\,(e_s - e_z)$

Conductive Heat Flux

- Computed from evaporative flux using Bowen Ratio

$$\phi_c = R_b \phi_e$$

$$R_b = C_b \frac{(T_s - T_z)}{(e_s - e_z)}$$

C_b =0.61 mb/^{o}C;

Summary

$$\phi_n = \underbrace{\phi_{sn} + \phi_{an} - \underbrace{\phi_{br} - \phi_e - \phi_c}_{\text{functions of } T_s}}_{\substack{\text{functions of external factors (met} \\ \text{and astronomical conditions)}}}$$

Strategies for computation: table look up

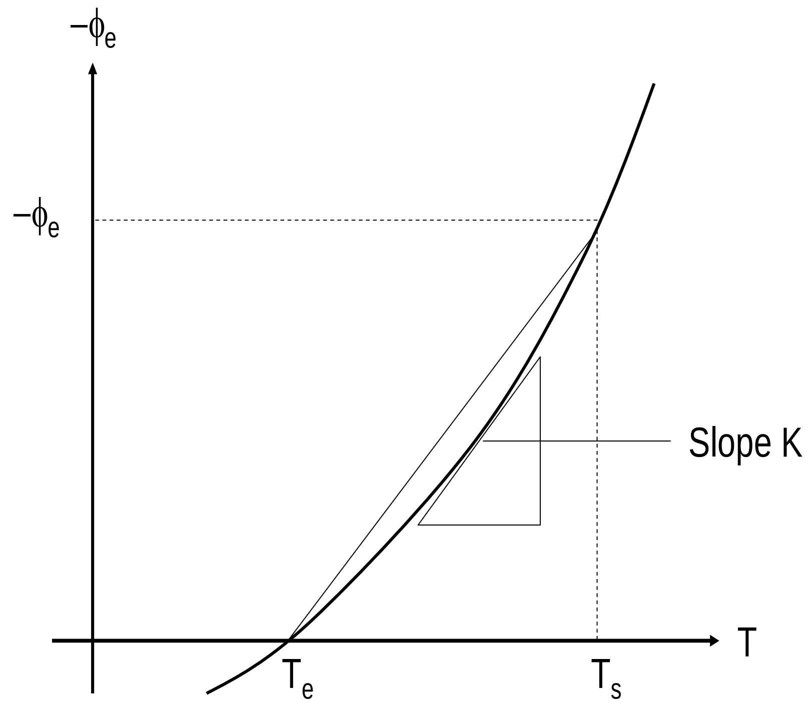
Self regulation: errors in calculations compensate

Linear Heat Transfer

- Equilibrium Temp, T_e
  - T_s for which $\phi_n = 0$
  - Function of met
- Surface Heat Exchange Coefficient, K
  - Slope of ϕ_n vs T_s

$$\phi_n = -K(T_s - T_e)$$

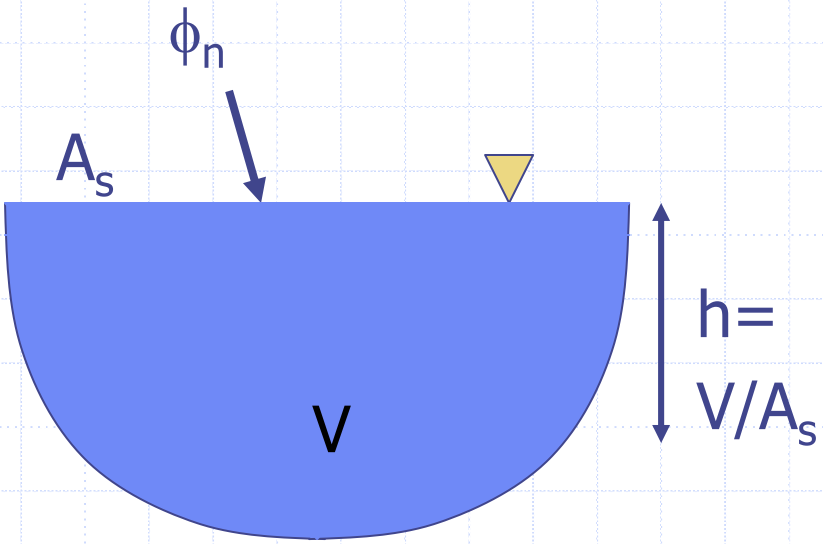
K ~ 20-50 $W/m^{2o}C$



Example: Periodic Heat Loss

$$\rho c V \frac{dT}{dt} = A_s \phi_n$$

$$\frac{dT}{dt} = -k(T - T_e)$$ $k = K/\rho c_p h$



$$T_e = \overline{T_e} + \Delta T_e e^{iwt}$$ $\omega = 2\pi/P$

$$T = \overline{T} + \Delta T * e^{i\omega t}$$

$$T = \overline{T_e} + \Delta T e^{i\theta} e^{i\omega t}$$

Periodic Heat Loss (cont'd)

$$T = \overline{T} + \Delta T * e^{i\omega t}$$

$$\Delta T* = \Delta T e^{i\theta}$$

$$T = \overline{T_e} + \Delta T e^{i\theta} e^{i\omega t}$$

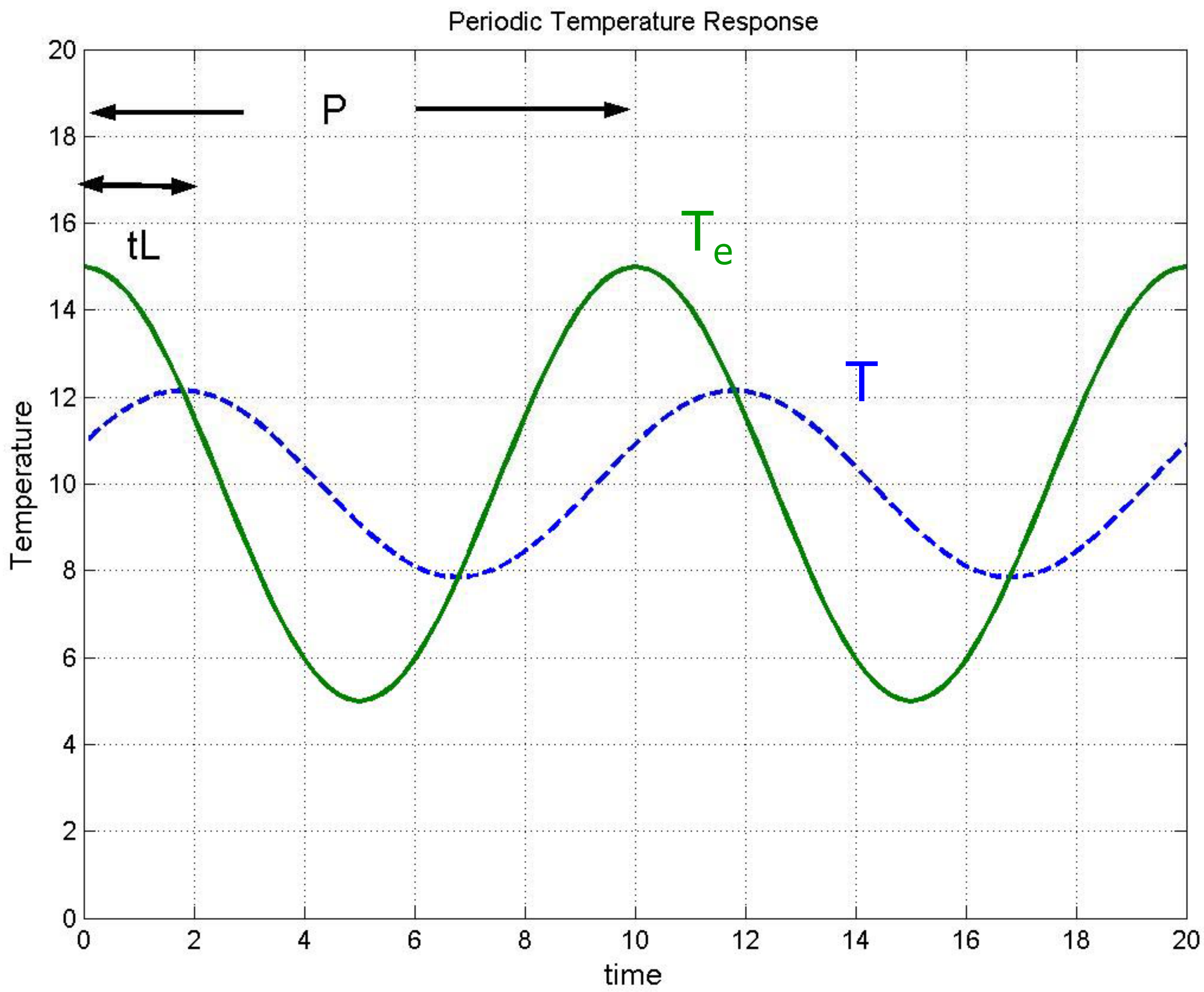
$$T = \overline{T_e} + \Delta T e^{i\omega(t - t_L)}$$

$$t_L = (\theta/2\pi)P$$

$$\frac{\Delta T *}{\Delta T_e} = \frac{k}{k + i\omega} = \underbrace{\frac{k}{\sqrt{k^2 + \omega^2}}}_{\text{Amplitude}} e^{\overbrace{i \tan^{-1}(-\omega/k)}^{\text{Phase lag}}}$$

$$\theta = \tan^{-1}(-\omega/k)$$

$$t_L = \frac{P}{2\pi}\tan^{-1}(\omega/k)$$


Periodic Temperature Response
P
tL
Te
T
Temperature
time
0
2
4
6
8
10
12
14
16
18
20

Examples

$K/\rho c$ = 1m/d*; h = 10m, $k=K/\rho ch = 0.1 d^{-1}$

P	1 day	365
$\omega=2\pi/P$	$0.17 d^{-1}$	$6.28 d^{-1}$
$\Delta T/\Delta T_e = k/(k^2+\omega^2)^{0.5}$	0.016	0.986
$\theta=\tan^{-1}(-\omega/k)$	-89^o	-10^o
$t_L = P/2\pi \tan^{-1}(\omega/k)$	0.247 d	10 d

* K ~ 48 $W/m^{2o}C$

Cooling Lakes and Ponds

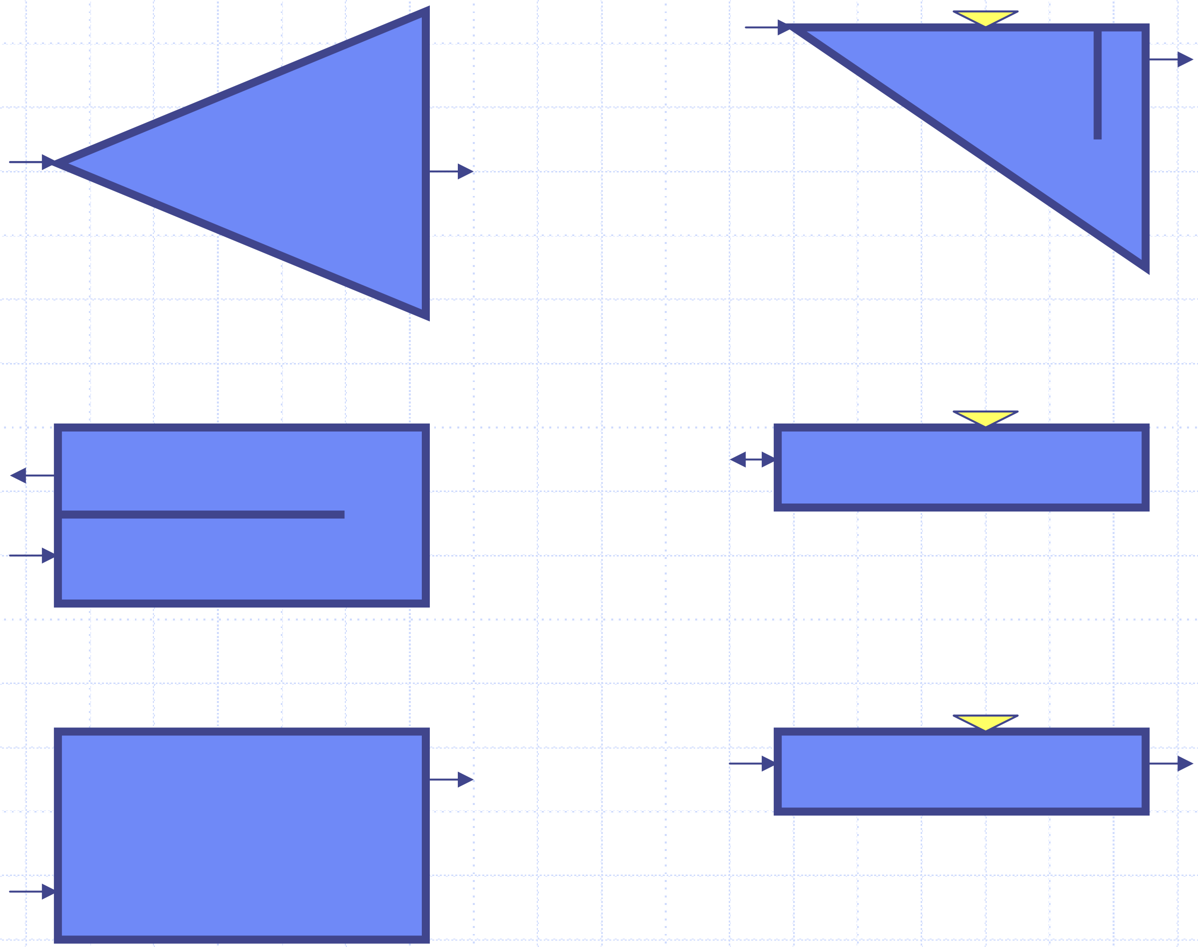
- Used to cool electric power plants
- Shallow (vertically well-mixed)
  - Erected with dikes
  - $T = T(x,y) + T(t)$
- Deep reservoirs
  - Damming of reservoirs
- Cooling capacity
  - $r = KA_p/\rho c Q_o$

Cooling Ponds

Deep Stratified

Shallow Dispersive

Shallow Recirculating



Plan View

Elevation

Example: shallow-longitudinal dispersive

$$\rho c Q_o \frac{dT}{dx} = \rho c W H E_L \frac{d^2T}{dx^2} - K(T - T_E)W$$

Single pass

$$\frac{T_i - T_E}{T_o - T_E} = \frac{4a e^{1/2E_L^*}}{(1+a)^2 e^{a/2E_L^*} - (1-a)^2 e^{-a/2E_L^*}}$$

Continuous operation

($T_o = T_i + \Delta T_o$)

$$\frac{T_i - T_E}{\Delta T_o} = \frac{4a e^{1/2E_L^*}}{(1+a)^2 e^{a/2E_L^*} - (1-a)^2 e^{-a/2E_L^*} - 4a e^{1/2E_L^*}}$$

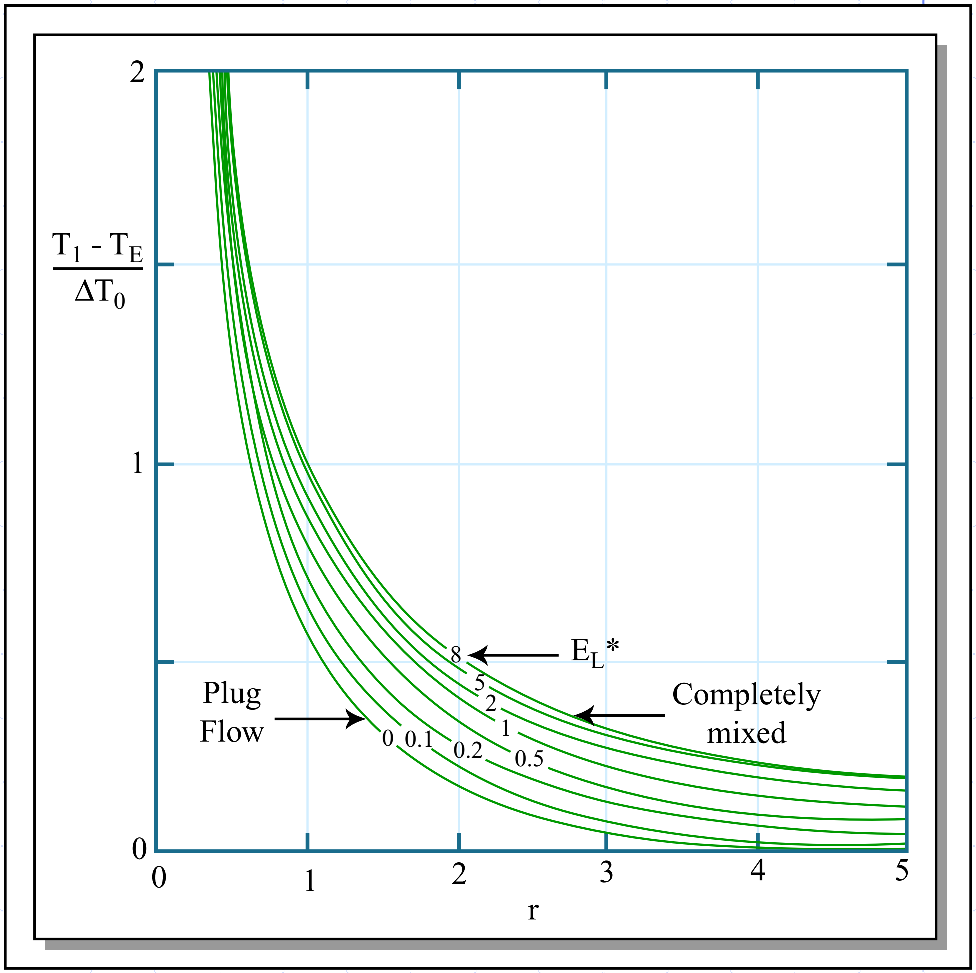


Figure by MIT OCW.

Jirka et al. (1978)

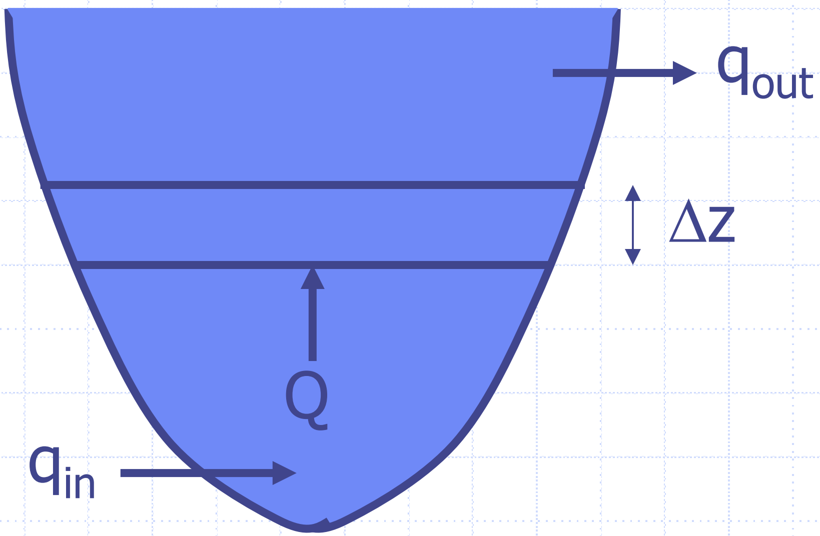
Stratification in Lakes & Reservoirs

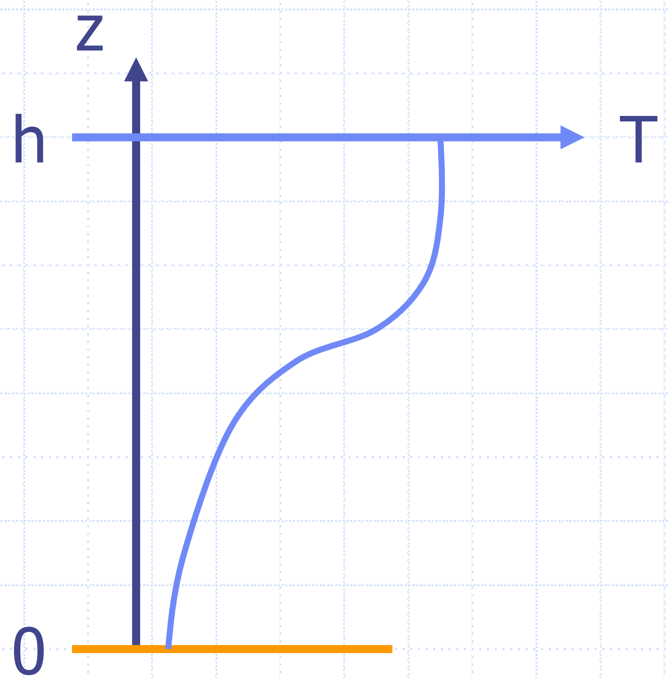
- Factors causing vertical stratification
  - Differential absorbtion
  - Reduced vertical mixing
- Factors causing horizontal stratification
  - Strong through flow
  - Strong wind
  - Differential absorbtion

Reservoir classification based on horizontal through flow (Orlob, 1969)

- Through flow velocity = L/(V/Q)
- Int'l wave speed ~ $(g\Delta\rho/\rho h)^{0.5}$ ~ Nh
  - N = buoyancy freq = $[(g/\rho)(d\rho/dz)]^{0.5}$
  - L = length; Q = flow; h = depth; V = vol
- F_r = LQ/VNh
  - $F_r \ll 1$ vertically stratified
  - $F_r \gg 1$ vertically mixed

1-D Reservoir Modeling



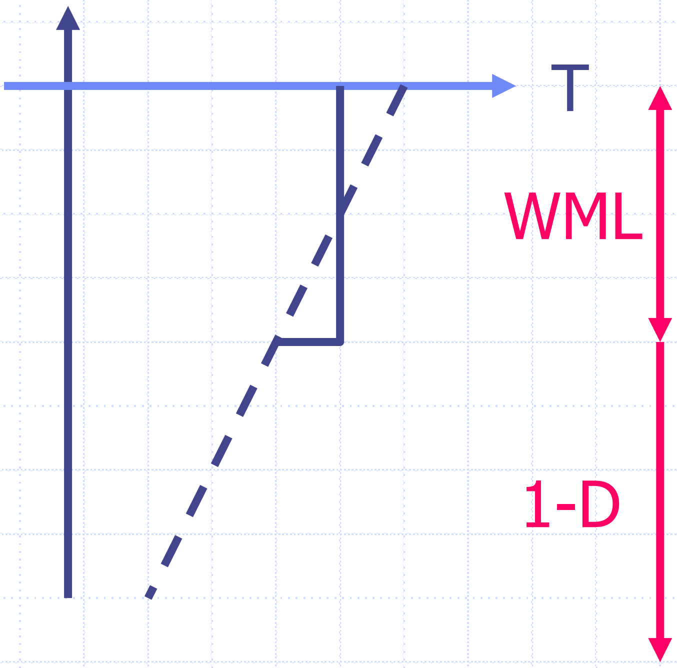


$$\frac{\partial Q}{\partial z} = q_{in} - q_{out}$$

$$\frac{\partial T}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}(QT) = \frac{1}{A}\frac{\partial}{\partial z}\left[AE_z\frac{\partial T}{\partial z}\right] + \frac{q_{in}T_{in} - q_{out}T}{A}$$

Surface Layer

- Well mixed layer
  - Convective mixing
  - Wind mixing
- Wind mixing algorithm for surface
  - Oceans (Kraus-Turner)
- 1-D model below



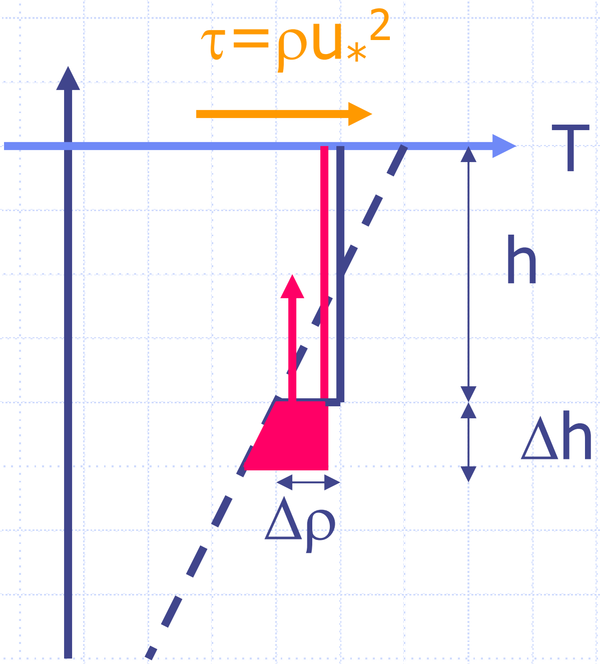
Surface Layer (cont'd)

$$\frac{\Delta PE}{A} = (\Delta\rho g \Delta h)\frac{h}{2} = \frac{\Delta\rho g h}{2} u_e \Delta t$$

$$\frac{\Delta KE}{A} = \rho u_*^2 u_s \Delta t \sim \rho u_*^3 \Delta t$$

$$\frac{\Delta PE}{\Delta KE} = const$$

$$\frac{u_e}{u_*} \sim \frac{u_*^2}{\frac{\Delta\rho}{\rho} gh} = Ri^{-1}$$



Many variants

Lake stability

Stability index (PE of water body with equivalent mass and heat content but uniform density – PE of stratified body)

$$SI = \int_0^h [\bar{\rho} - \rho(z)][z - z_c] g A(z) dz$$

$$\bar{\rho} = \int_0^h \rho(z) A(z) dz \Big/ \int_0^h A(z) dz$$ Average density

$$z_c = \int_0^h \rho(z) A(z) z dz \Big/ \int_0^h \rho(z) A(z) dz$$ Center of mass