Lecture #3: <u>Two-Slit Experiment. Quantum Weirdness</u>

Last time:

1. Wave character of e^-
   X-ray and e^- diffraction using atom-spacings in a crystal as ruler to <u>measure</u> $\lambda(p)$. Find $\lambda = h/p$.
   de Broglie hypothesis: $\lambda = \dfrac{h}{p}$ for all particles
   Questions: what happens when λ is comparable to the size of the container, or λ^3 is large compared to the volume available to each atom: V/N?

2. Rutherford — postulated (based on Geiger-Marsden experiments) planetary atom in order to "solve" the space-filling requirement. But:
   * no quantization
   * radiative collapse

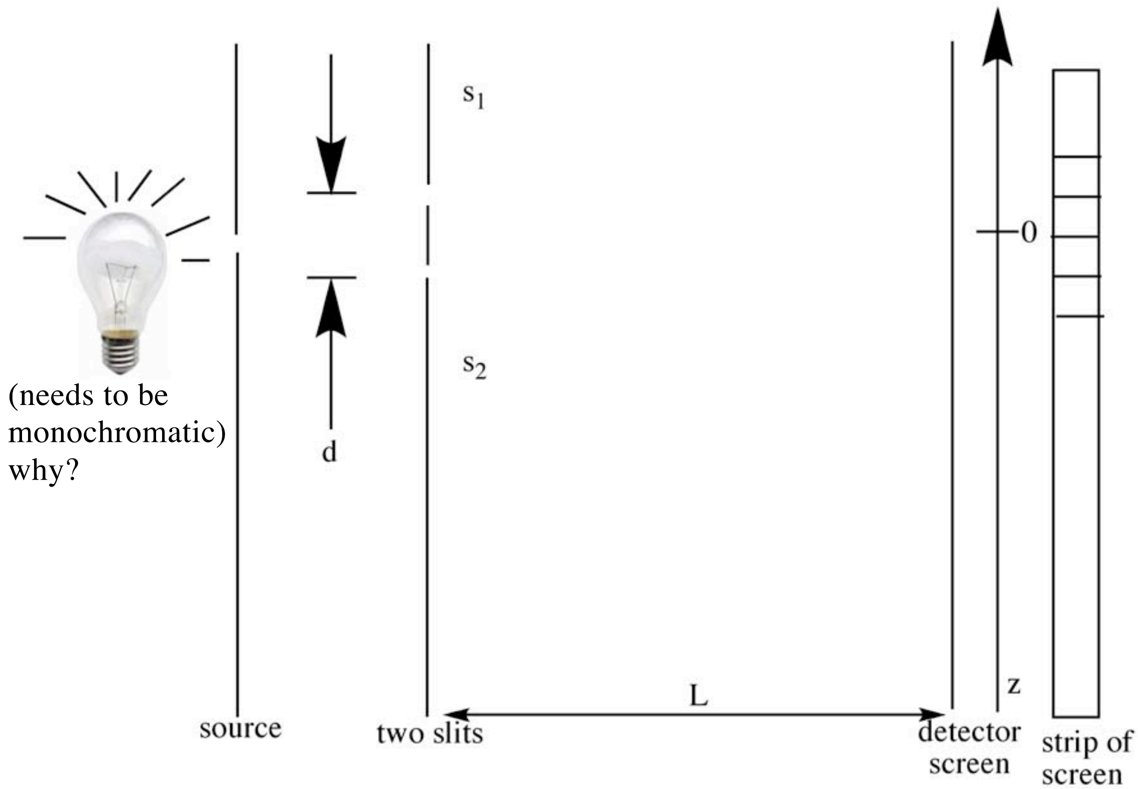
3. Bohr $\vec{\ell} = \vec{r} \times \vec{p} = |r||p| \equiv \hbar n$ n = 1, 2, 3… (quantization of ℓ!)
   de Broglie: $\underbrace{2\pi r_n}_{\text{circumference of Bohr orbit}} = n\lambda_n$
   Two *ad hoc* hypotheses to "prevent" radiative collapse. Leads to requirement of quantized energy levels.

4. Idea (Ritz, Balmer, Rydberg) that spectral lines are transitions between quantized energy levels. "Explains" spectra of $1e^-$ atoms.
   * nothing about radiative lifetimes or relative transition strengths
   * effect of magnetic fields (transition line splits into too many components $\rightarrow$ e^- spin)
   * not a clue to explain spectrum of $2e^-$ atom: Helium.

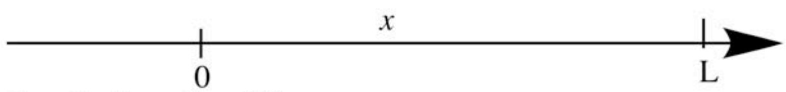
Today:

2-slit experiment.
* interference
* taste of quantum measurement theory
* qualitative stuff about waves
* glimpse of uncertainty principle

This stuff is weird!



distance to screen: $L \gg d$



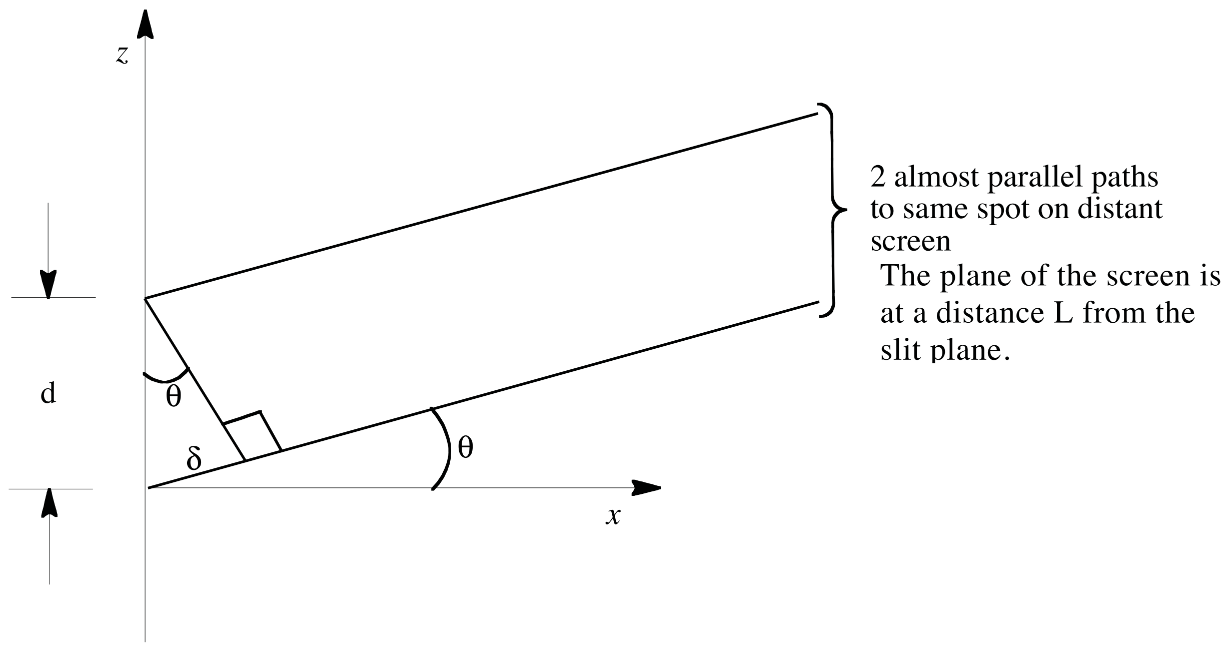
Width of each slit $\ll d$. Choose the width to be $\sim\lambda$ and $d > 10\lambda$.

We expect to see an interference pattern when both slits are open but no interference when one of the slits is covered.

Constructive interference results when the paths from s_1 and s_2 to *same point* on the screen differ by an integer multiple of λ.

Call the direction along the screen z and the direction along the $\perp$ path from slits to screen x.

Here is a blow-up of the region near the 2 slits



because L ≫ d, can treat the two rays as parallel yet intersecting at the same point on the screen.

For *constructive* interference it is necessary that the two paths differ in length by $\delta = n\lambda$

$$\delta = d \sin \theta \qquad \text{(see diagram)}$$

so we get a set of θ-values at which constructive interference occurs

$$n\lambda = d \sin \theta$$

$$\theta_n = \sin^{-1} \frac{n\lambda}{d} \qquad \theta_0 = 0 \text{ (central spot for n = 0)}$$

in the *small θ limit*, $\theta_n \approx \dfrac{n\lambda}{d}$ $\qquad (d \gg \lambda)$

See a series of equally spaced bright regions (constructive interference) separated by dark regions (destructive interference).

On the screen, the bright regions are at $z = 0, \pm L \sin\theta_n \approx \pm \dfrac{L}{d} n\lambda$.

OK. Now what happens if we cover one of the slits?

Interference pattern disappears. [Width of central bright zone broadened by diffraction.] Does the pattern on the screen tell us which slit was covered? How? Are we allowed to know?

Yes. Asymmetry What does this mean?

Suppose we reduce the intensity of light entering the 2-slit apparatus so much that, at any given time there is either 0 or 1 photon in the apparatus. It is rather straightforward to measure the intensity and know that the intensity is small enough to satisfy this requirement. What do you need to know to compute the < 1 photon at a time intensity?

What will we see?

* no interference pattern?
* weak interference <u>on top of a constant background</u>, which suggests that only rare fluctuation events yield 2 photons simultaneously traversing the apparatus?
* the usual, full 100% modulated interference pattern?

We expect the intensity distribution to exhibit interference, based on the wave nature of light. But we know that light also has particle characteristics.

What do we see on a 2-D detector with single-event sensitivity and time resolution?

The continuous distribution "collapses" into localized single events. Each event is <u>independent</u> of all other events and one point cannot resemble a distribution.

Reconcile?

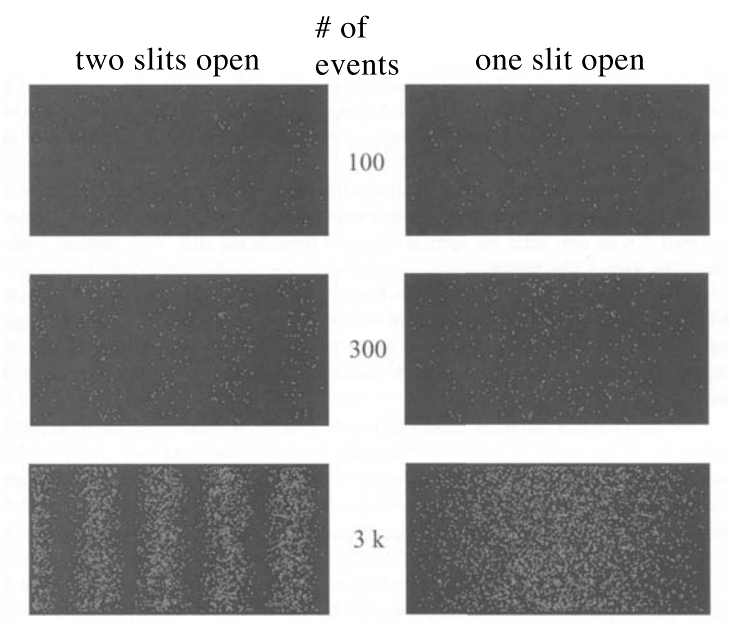


FIGURE 1.18
When a particle passes through the two-slit screen in Figure 1.14, its arrival at the second screen is recorded by a dot. As more and more particles arrive at the screen, an interference pattern slowly builds up. The three panels on the left record the arrival of 100, 300, and 3000 particles. The three panels on the right show the arrival of the same number of particles as those on the left, but one of the slits is closed, yielding no interference pattern. These figures are computer simulations of actual experiments carried out by A.Tomomura et al., *Am. J. Phys.*, **57**, 117 (1980).

See one-photon event as a dot on the screen. Initially the dots look randomly distributed. Eventually, once a sufficient number of dots has accumulated, the interference pattern emerges. You should be *amazed* by this!

The interference pattern goes away when either slit is covered. (What happens when we use white light instead of monochromatic light?)

Quantum Weirdness

You are not allowed to know *which one* of 2 open slits a single photon went through. The interference is at the single-event level, not at the many-event level. The interference is of *one photon with itself*, not with another photon. Is this weird or what?

We need to describe the two-slit experiment by some sort of probability amplitude distribution and to describe an experiment as the sum of interfering amplitudes followed by some sort of operation that expresses the action of the detector (i.e. collapse each one-photon signal to a single spot).

Looking ahead: Light follows a wave equation. The probability amplitude will look like this:

$$u(x,t) = A \sin(kx - \omega t)$$

If $u(x,t)$ is a probability amplitude, what is the probability density? What is the difference between a probability amplitude and a probability density? Can either one be negative at some values of x and t?

<u>Wavelength</u>

$u(x + \lambda, t) = u(x,t)$ (spatial repeat distance)

$A \sin[kx + k\lambda - \omega t] = A \sin[kx - \omega t]$

if $k\lambda = 2\pi$

$\boxed{k = \frac{2\pi}{\lambda}}$ "wave number" $\lambda = \frac{h}{p} \rightarrow k = \frac{2\pi}{h} p = p/\hbar$

(in 3-D, $\vec{k}$ points in the direction of wave motion. Large k implies small λ and large p.) k is 2π times the number of wavelengths per unit length.

<u>Velocity</u>:

Take a snapshot of a wave in time.

How does the *phase point*, x_ϕ, move?

$$\phi = kx_\phi - \omega t \qquad , \qquad \text{pick } \phi = 0$$

$$x_\phi = \frac{\omega t}{k}$$

$$\frac{dx_\phi}{dt} = \frac{\omega}{k} \qquad \text{velocity of phase point}$$

For a wave of the form $u(x,t) \propto \sin(kx - \omega t)$

velocity is $+\dfrac{\omega}{k}$ (moving in $+x$ direction)

not too surprising

$$\omega = 2\pi\nu$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda \underbrace{\Rightarrow}_{\text{for light}} \nu = c/\lambda!$$

$\omega/k = c$ (for monochromatic electromagnetic radiation propagating in vacuum).

Intensity of an electromagnetic wave is $\propto \varepsilon^2$

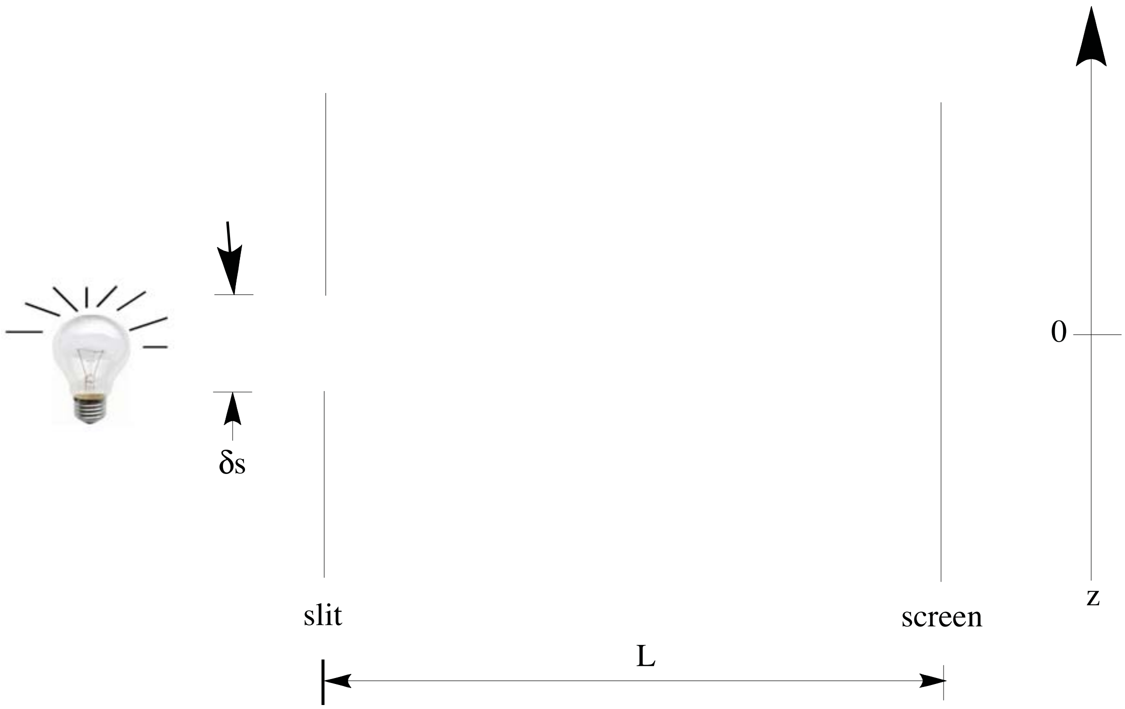
$$I(x,t) = \left|\sum_i a_i u\left(k_i x + \omega_i t\right)\right|^2 \qquad \text{superposition of amplitudes}$$

Sum first, then square. amplitude of i^{th} component

A taste of the uncertainty principle

Suppose we want to spatially localize a particle.

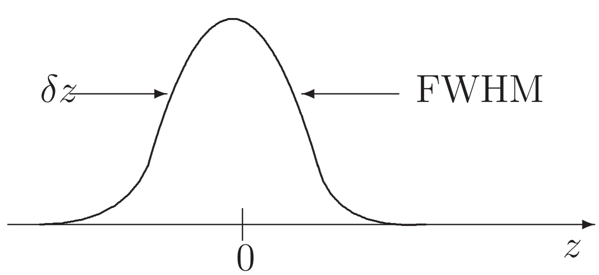
Put it through a slit of width $\delta s = \delta z$



The wave nature of the particle $\lambda = \dfrac{h}{p}$ implies that there will be "diffraction" of the particle by the slit. This results in spreading of the image of the slit on the screen.

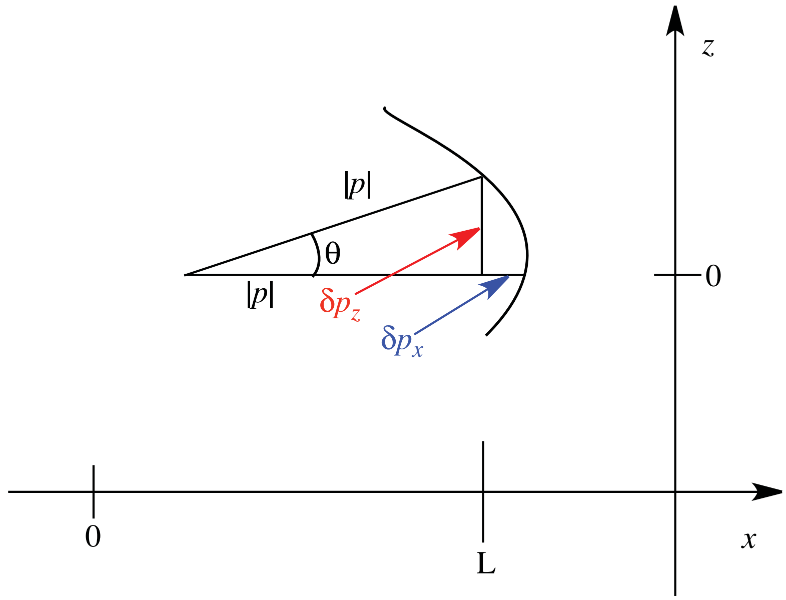
Use the same algebra as for the 2 slit experiment. Paths from *top and bottom edges of the slit* to a point z on the screen must differ by $\lambda/2$ to get *destructive* interference. [Interference is less complete from points not at edges of slit.]

This means that



$$\delta z = 2\left(\frac{\lambda}{\delta s} L\right)$$

This means that p_z is uncertain because $\vec{p}$ is a vector quantity. ($|\vec{p}|$ is conserved, but the angular uncertainty results in a magnitude uncertainty of p_z.)



Photon that hits at center has $p_x = |p|$, $p_z = 0$.

Photon that hits at edge has smaller p_x, larger p_z.

$$\delta p_z \approx |p| \frac{\lambda}{\delta s} \qquad p = \frac{h}{\lambda}$$

$$\approx \frac{h}{\lambda}\frac{\lambda}{\delta s} = \frac{h}{\delta s}$$

uncertainty in p_z resulting from slit of width δs.

$\delta z \delta p_z \approx h$ is an uncertainty principle. An attempt to restrict position (δz) results in uncertainty in p_z (δp_z). QM is based on what could, in principle, be measured. Every experiment must be analyzed in this way.

<u>Today</u>:
* 2-slit experiment. Can't know which slit. Photon interferes with itself.
* waves: c, λ, k
* amplitude and intensity
* uncertainty of joint measurement of x and p_x.

Next Lecture: classical wave equation in preparation for Schrödinger Equation.
Read Chapter 2 before next lecture!