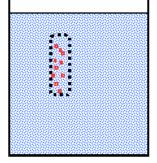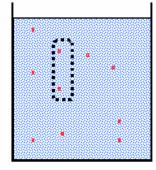BE.011/2.772
Problem Set 3
Due March 3, 2004

1. Dialysis is common laboratory procedure used to remove low molecular weight solutes from protein solutions. Example situations include: removal of removal of free, unreacted label after fluorescently labeling a protein; and removal of urea from a denatured protein solution to allow refolding of the protein. The protein solution is sealed inside a dialysis bag or tubing and placed in container with a large excess of pure water, as shown in the figure. The system is then allowed to come to equilibrium. For this problem, consider an idealized system where the only solute in the dialysis bag is the low molecular weight solute (i.e., no protein is actually present.)

   (a) Explain why the combined system of the protein solution (system "A") and the water (system "B") may be considered to be an <u>isolated</u> system.
   (b) Use a lattice model, with each individual water molecule occupying a single site in the lattice, to show that the entropy of System B is zero at the start of the experiment. Designate the number of water molecules N_{WB} and the number of total lattice sites M_B.
   (c) Use a lattice model to express the entropy of system A at the start of the experiment in terms of the number of water molecules in system A, N_{wA}, the number of solute particles, N_s, and the total number of lattice sites in system A, M_A. For your final expression, use an appropriate approximation to eliminate factorial expressions. You may assume that the water molecules and solute molecules are comparable in size.

Initial state

Equilibrium state

   (d) Now write an expression for the entropy of the combined system under the conditions where water and solute can freely pass through the dialysis membrane. For every solute molecule that crosses over to system B, a water molecule must cross over to system A. (You may want to keep track of the number of solute molecules by noting that $N_S = N_{SA} + N_{SB}$ and the number of water molecules by noting that $N_{WA} + N_{WB} = N_W$).
   (e) Show that you can write the entropy expression in terms of one system variable (i.e., one parameter that changes value during the progress of the experiment), and briefly describe how to determine the equilibrium condition in terms of that variable and constants in the system. [You do not need to work out the equilibrium condition (lots of algebra involved) but feel free to predict what it is if you like.]
   (f) Is work done in the approach to equilibrium? Explain why or why not.
   (g) Does the internal energy U of the system change? Explain why or why not.