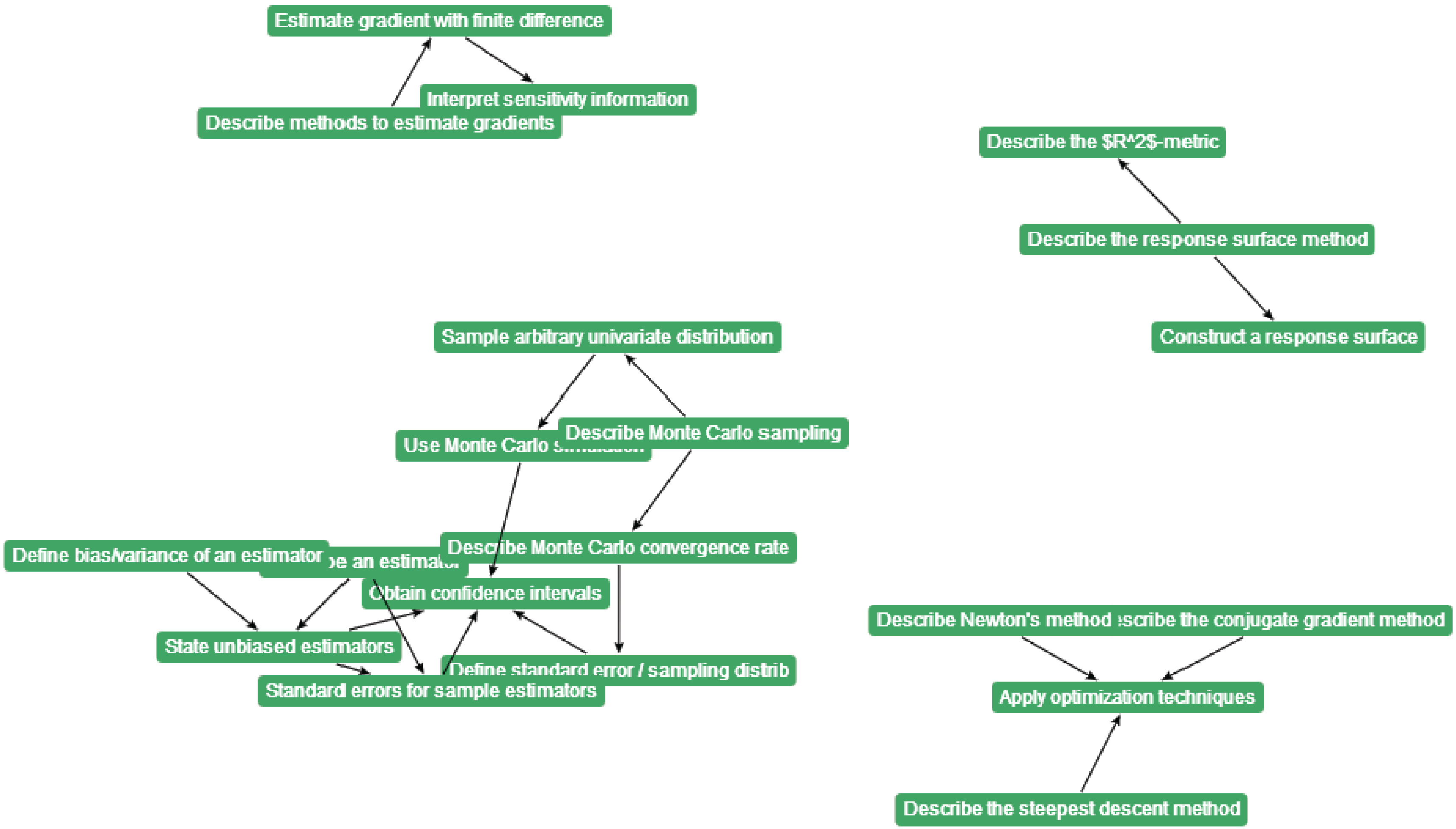
Probabilistic Analysis and Optimization

- In this last module we will cover:
  - Probabilistic analysis via Monte Carlo simulation
  - Monte Carlo convergence and error estimation
  - Statistical sampling
  - Design of experiments
  - Sensitivity analysis
  - Introduction to design optimization

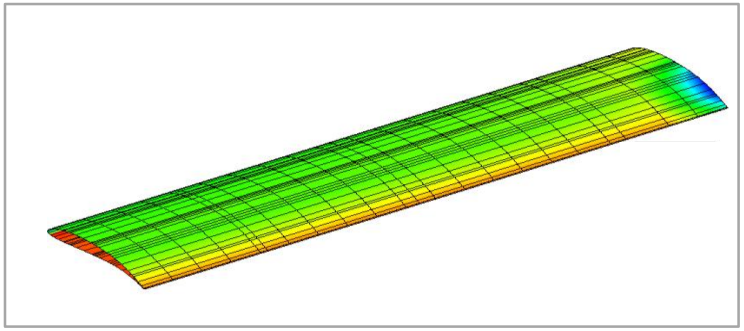
Measurable Outcomes



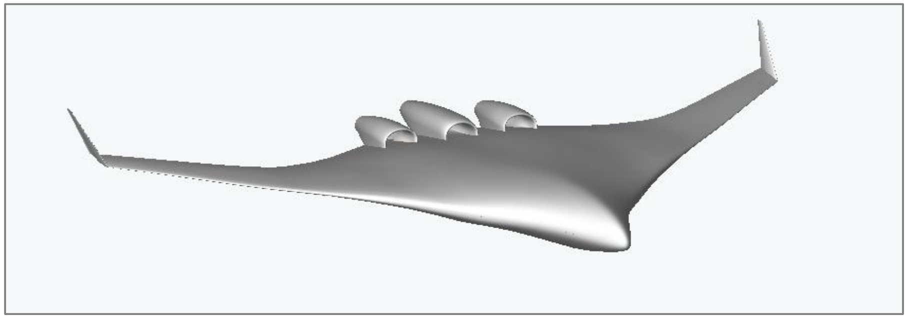
Today’s Topics

1. Importance of probabilistic analysis in aerospace design
2. Monte Carlo (MC) methods
3. Probability & statistics refresher
4. Turbine blade heat transfer example
5. MC method for uniform distributions
6. MC method for non-uniform distributions

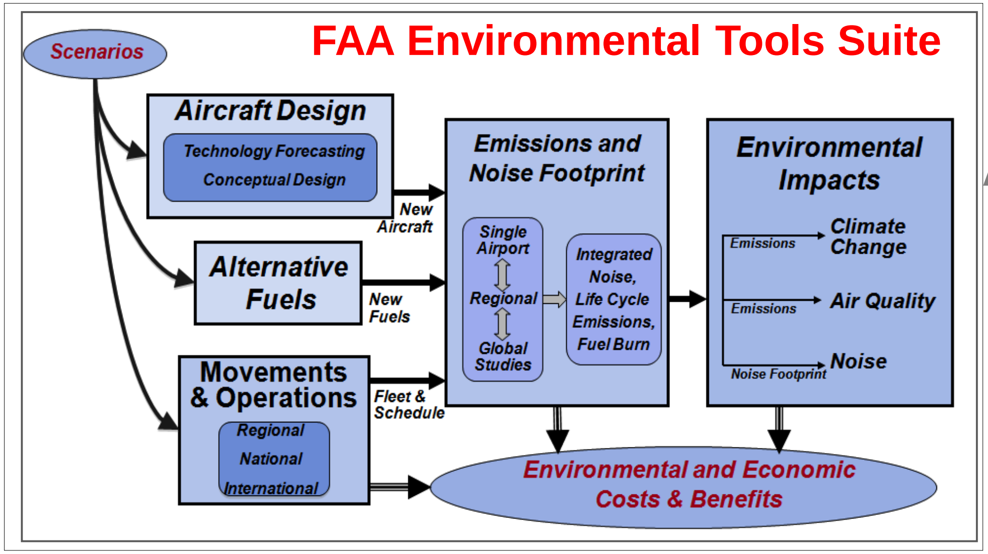
Design, optimization and decision under uncertainty



Aerostructural shape optimization



Vehicle system optimization



Aviation environmental policy decision-making

Characterizing, representing and analyzing uncertainty in simulation tools is essential for aerospace systems

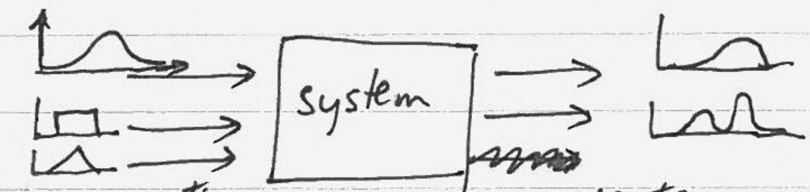
- To support decision-making (optimization, control, design, policy)
- To inform model development

2. Monte Carlo methods

2. We will study Monte Carlo method
 - a way of determining how uncertainty in inputs translates to uncertainty in outputs

Name: coined by researchers at Los Alamos as a joke: Casino where one's uncle used to gamble.

Most famous early use: Enrico Fermi 1930 use to compute properties of newly discovered neutron.



etc.

inputs (e.g. BCs, ICs, forcing terms)

outputs (e.g. temperature, velocity, lift, drag etc.)

General steps in MC methods: 1. Define distributions of inputs

2. Sample inputs randomly, run deterministic solve on each one

3. ~~Use~~ Analyze resulting distributions of outputs to estimate desired statistical outputs

3. Probability & statistics refresher

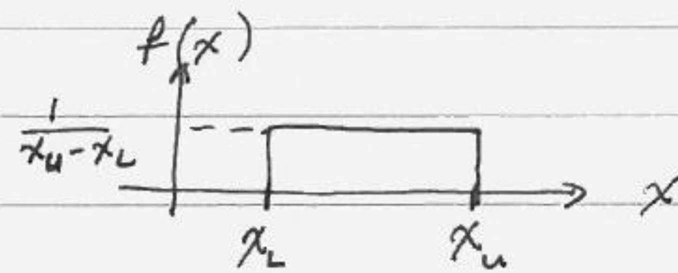
PDF: If x is a random variable, then

$$P\{a \le x \le b\} = \int_a^b f(x)\,dx$$

probability that x lies between a & b

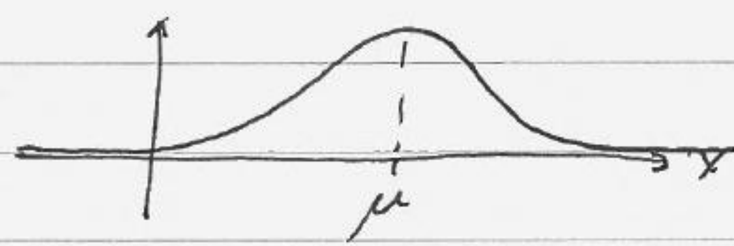
PDF of x

e.g. uniform



$$f(x) = \begin{cases} \frac{1}{x_u - x_L}, & a \le x \le b \\ 0, & \text{otherw} \end{cases}$$

e.g. Normal



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

mean μ, s.d. σ

$x = N(\mu; \sigma)$

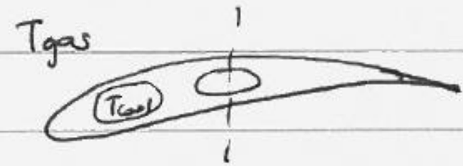
4. Turbine blade heat transfer example

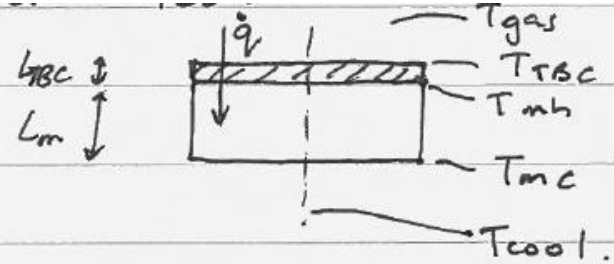
T → temperature

k → thermal cond.

h → convective heat xfer coeff.

L → thickness





Inputs: h_{gas}, T_{gas}, k_{TBC}, L_{TBC}, k_m, L_m, h_{cool}, T_{cool}.

Eqs. (15.1–4) solve to determine T_{TBC}, T_{mh}, T_{mc}, $\dot{q}$

Output of interest; T_{mh}, hot-side metal temp.

Deterministic calculation: Ex. 15.1 ⇒ $T_{mh} = 1121.8$ K

We will consider impact of variability in L_{TBC} on T_{mh}.

Turbine blade heat transfer example

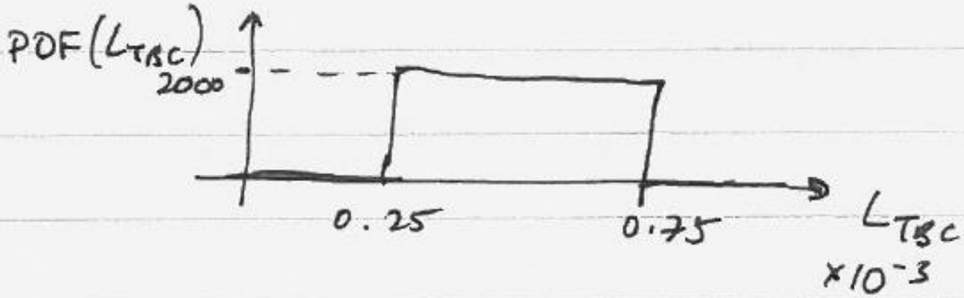
In our MC approach we will:

1. Define distribution fof L_{TBC} (knowledge of manufacturing process)
2. Sample randomly from our L_{TBC} distribution, analyze (deterministically) each case in the sample
3. Collect the value of T_{mh} for each case run. Use these T_{mh} data to estimate e.g.
   - → distribution of T_{mh} would be observed in population of infred blades
   - → Prop. that T_{mh} exceeds some critical value
   - → Mean value of T_{mh}, $\mu_{T_{mh}}$
   - → std. dev. of T_{mh}, $\sigma_{T_{mh}}$

5. MC method for uniform distributions

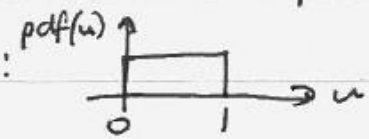
step1: Assume L_{TBC} is uniformly distributed from

$0.00025\,m < L_{TBC} < 0.00075\,m$



step2: (a) Sample. To generate random numbers in Matlab use "rand"

→ returns u:



→ then we can use $L_{TBC} = 0.00025 + 0.0005u$

Simulation Challenge

- You are given the model in the file "blade1D.m"
  - function [Ttbc, Tmh, Tmc, q] = blade1D(hgas, Tgas, ktbc, Ltbc, km, Lm, hcool, Tcool)
- Write a Monte Carlo simulator
- Run these cases with N=10, 100, 1000:
  - LTBC ~ U(0.00025,0.00075)
  - Other variables deterministic at values given in HW8

  Hint: the Matlab function rand returns a uniformly distributed random number on the interval (0,1).
- Generate an output histogram for Tmh and estimate the output mean
- Qualitatively, what evidence is there that your MCS implementation is correct?

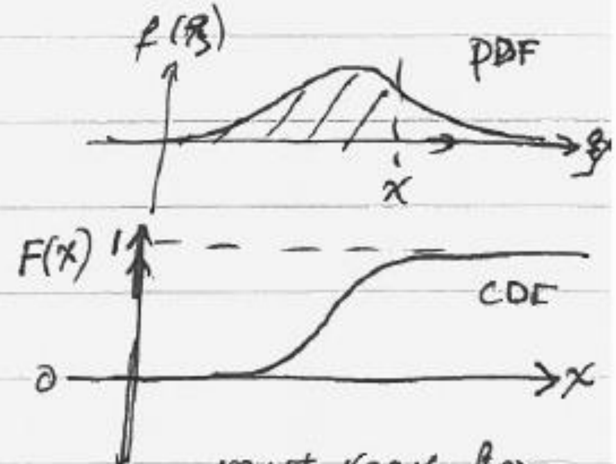
6. MC method for non-uniform distributions

- How to generate random no.s w/ non-uniform dist$^{n}_{s}$?

⇒ we use inversion method.

CDF: $$\underbrace{F(x)}_{\text{CDF}} = \int_{-\infty}^{x} \underbrace{f(\xi)}_{\text{PDF}}\, d\xi$$

$$F(x) = P\{X \le x\}$$

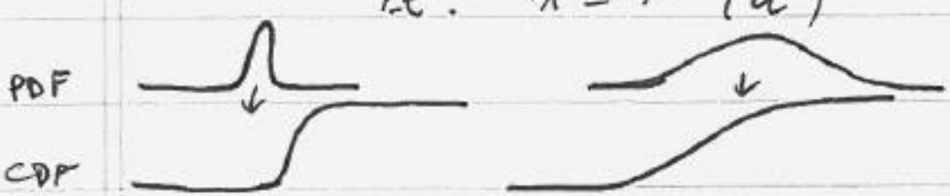


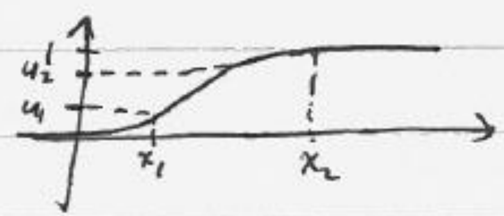
must range from 0 to 1.

Inversion method:

1. Generate u from $U(0,1)$
2. Given u, find value of x at which $u = F(x)$

   i.e. $x = F^{-1}(u)$





Triangular distributions