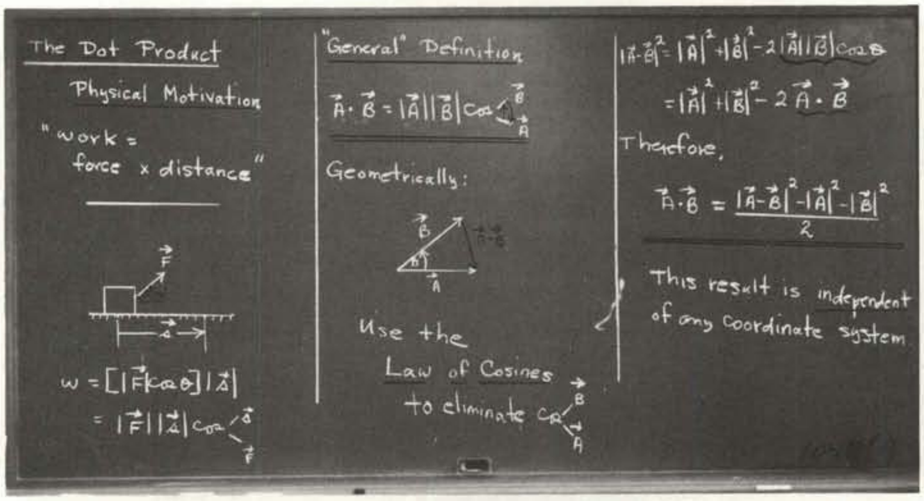
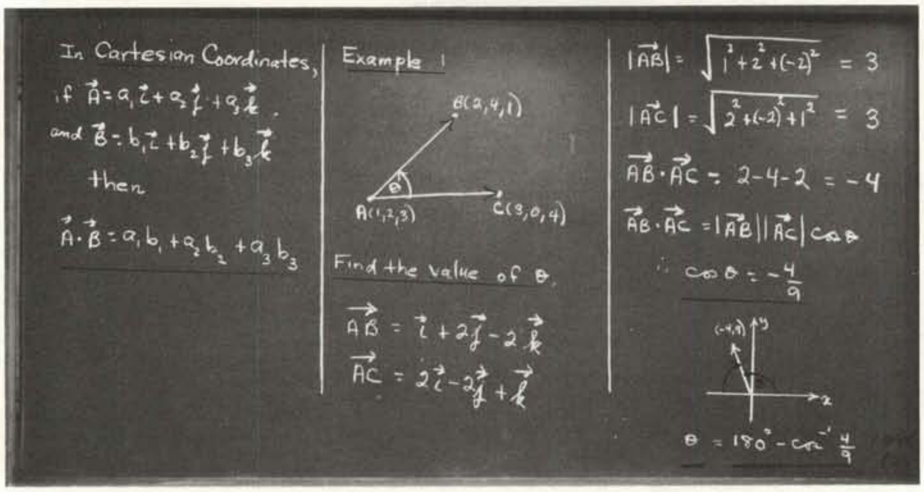
## Unit 4: The Dot Product

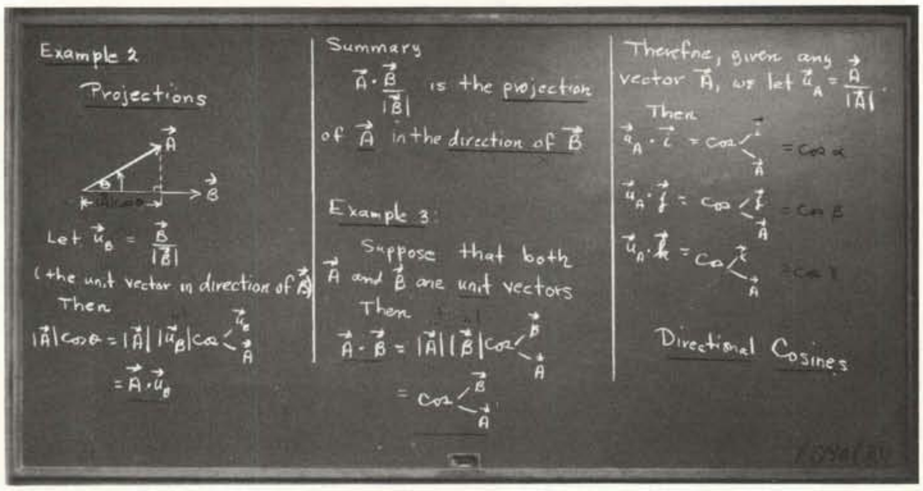
1. Lecture 1.040




a.*



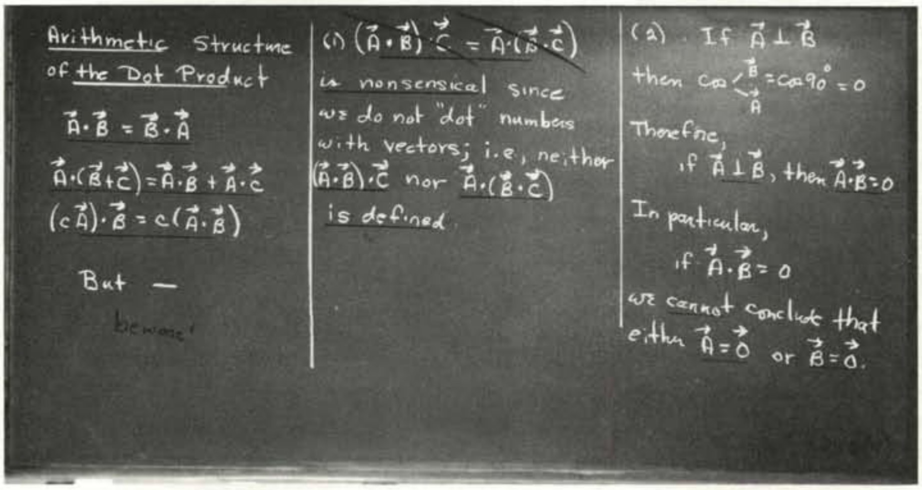

b.




c.

*See note on next page.

Lecture 1.040 continued




d.

<u>Note</u>: Last equation on first board (a) should read:

$$\vec{A}\cdot\vec{B} = \frac{|\vec{A}|^2 + |\vec{B}|^2 - |\vec{A} - \vec{B}|^2}{2}$$

2. Read Thomas 12.6.

3. Exercises:

1.4.1(L)

a. Let $\vec{A} = \vec{i} + \vec{j} + \vec{k}$ and let $\vec{B} = 2\vec{i} + 3\vec{j} + 4\vec{k}$. Find the relationship between x, y, and z if $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$, where $\vec{C} = x\vec{i} + y\vec{j} + z\vec{k}$.

b. Use vector arithmetic to generalize the result of part (a) by showing that if $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$ and we know that $\vec{A} \neq \vec{0}$ and $\vec{B} \neq \vec{C}$, then $\vec{A}$ is perpendicular to $\vec{B} - \vec{C}$.

c. With $\vec{A}$ and $\vec{B}$ as in part (a), use the result of part (b) to interpret geometrically the set of all vectors $\vec{C}$ for which $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$.

d. In terms of structure, explain why the result of this exercise is not a contradiction of the cancellation law of numerical arithmetic.

1.4.2

a. Let $P_o(2,3,4)$ be in the plane M and let $\vec{V} = 5\vec{i} + 8\vec{j} + 6\vec{k}$ be perpendicular to M. Determine the relationship between x, y, and z if P(x,y,z) is to lie in the plane M. (This relation is called the equation of the plane in Cartesian coordinates.)

b. Is (1,1,8) in the plane M? If not, where is it relative to the plane? Explain.

1.4.3

Lines are drawn from A(1,2,3) to both B(3,3,5) and C(4,4,9). Determine the measure of $\measuredangle$ BAC (i.e., the angle between $\vec{AB}$ and $\vec{AC}$).

1.4.4(L)

Find the point at which the line through A(2,3,4) perpendicular to the line $y = -3x + 2$ meets the line $y = -3x + 2$.

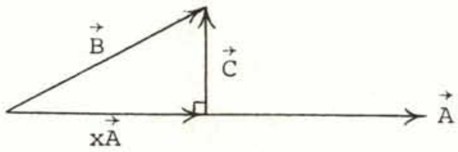
1.4.5(L)

Let $\vec{v}_1 = 3\vec{i} + 2\vec{j} + 5\vec{k}$ and let $\vec{v}_2 = 2\vec{i} + 7\vec{j} + 3\vec{k}$. Determine the value of $(3\vec{v}_1 + 4\vec{v}_2) \cdot (4\vec{v}_1 - 3\vec{v}_2)$.

1.4.6(L)

a. In the diagram below, $\vec{A}$ and $\vec{B}$ are given vectors. Determine $x\vec{A}$ and $\vec{C}$ in terms of $\vec{A}$ and $\vec{B}$ by using vector arithmetic.

b. Do the same as in (a) but use geometry instead of arithmetic.




1.4.7

Find the length of the projection of $\vec{i} + 5\vec{j} + 6\vec{k}$ onto the vector $2\vec{i} + 6\vec{j} + 3\vec{k}$.

1.4.8(L)

Use vector methods to show that the perpendicular distance from the point $A(x_o,y_o)$ to the line $ax + by + c = 0$ is given by

$$\frac{|ax_o + by_o + c|}{\sqrt{a^2 + b^2}}$$

1.4.9

The line L is determined by the two points in space, (1,2,3) and (2,4,5). Find the perpendicular distance from the point (3,5,9) to L.