Quiz On The Day Before Lecture # 9

Arbitrage Pricing Theory

Suppose that there are two independent economic factors F1 and F2. The risk-free rate is 6%, and all stocks have independent firm-specific components with a standard deviation of 45%. The following are well-diversified portfolios:

| Portfolio | Beta on $F_1$ | Beta on $F_2$ | Expected Return |
|---|---|---|---|
| A | 1.5 | 2.0 | 31 |
| B | 2.2 | -0.2 | 27 |

Table 1: Scenarios for 2 stocks with 2 Factors

What is the expected return-beta relationship in this economy?

Solution
BKM ch. 11, p. 335 # 2

$$E\left(r_p\right) = r_f + \beta_{1,p}\left[E\left(r_1\right) - r_f\right] + \beta_{2,p}\left[E\left(r_2\right) - r_f\right]$$

We need to find the risk premium [rp] for each of the two factors:
$rp_1 = [E(r_1)-r_f]$ and
$rp_2 = [E(r_2)-r_f]$

To do so, the following system of two equations with to unknowns must be solved:
$21 = 6 + 1.5 \text{ x } rp_1 + 2.0 \text{ x } rp_2$
$27 = 6 + 2.2 \text{ x } rp_1 + (-0.2) \text{ x } rp_2$

The solution to this set of equation is

$rp_1 = 10\%$ and $rp_2 = 5\%$

Thus, the expected return-beta relationship is:

$$E\left(r_p\right) = 6\% + \beta_{1,p} \cdot 10\% + \beta_{2,p} \cdot 5\%$$