## Options (2)



Class 20
Financial Management, 15.414

## Today

## Options

- Option pricing
- Applications: Currency risk and convertible bonds


## Reading

- Brealey and Myers, Chapter 20, 21


## Options

Gives the holder the right to either buy (call option) or sell (put option) at a specified price.
$>$ Exercise, or strike, price
$>$ Expiration or maturity date
> American vs. European option
> In-the-money, at-the-money, or out-of-the-money

## Option payoffs (strike $=\mathbf{\$ 5 0}$ )






## Valuation

## Option pricing

How can we estimate the expected cashflows, and what is the appropriate discount rate?

## Two formulas

> Put-call parity
$>$ Black-Scholes formula*

* Fischer Black and Myron Scholes


## Put-call parity

## Relation between put and call prices

$$
\mathrm{P}+\mathrm{S}=\mathrm{C}+\mathrm{PV}(\mathrm{X})
$$

$\mathrm{S}=$ stock price
$\mathrm{P}=$ put price
C = call price
$\mathrm{X}=$ strike price
$\mathrm{PV}(\mathrm{X})=$ present value of $\$ \mathrm{X}=\mathrm{X} /(1+\mathrm{r})^{\mathrm{t}}$
$r=$ riskfree rate

## Option strategies: Stock + put





## Option strategies: Tbill + call





## Example

On Thursday, Cisco call options with a strike price of $\$ 20$ and an expiration date in October sold for $\$ 0.30$. The current price of Cisco is $\$ 17.83$. How much should put options with the same strike price and expiration date sell for?

## Put-call parity

$$
\begin{aligned}
& P=C+P V(X)-S \\
& C=\$ 0.30, \quad S=\$ 17.83, X=\$ 20.00 \\
& r=1 \% \text { annually } \rightarrow 0.15 \% \text { over the life of the option } \\
& \text { Put option }=\mathbf{0 . 3 0}+\mathbf{2 0} / 1.0015-17.83=\$ 2.44
\end{aligned}
$$

## Black-Scholes

## Price of a call option

$$
C=S \times N\left(d_{1}\right)-X e^{-r T} N\left(d_{2}\right)
$$

$$
\begin{aligned}
& S=\text { stock price } \\
& X=\text { strike price } \\
& r=\text { riskfree rate (annual, continuously compounded) } \\
& T=\text { time-to-maturity of the option, in years } \\
& d_{1}=\frac{\ln (S / X)+\left(r+\sigma^{2} / 2\right) T}{\sigma \sqrt{T}} \\
& d_{2}=d_{1}-\sigma \sqrt{T}
\end{aligned}
$$

$N(\cdot)=$ prob that a standard normal variable is less than $d_{1}$ or $d_{2}$ $\sigma=$ annual standard deviation of the stock return

## Cumulative Normal Distribution



## Example

The CBOE trades Cisco call options. The options have a strike price of $\$ 20$ and expire in 2 months. If Cisco's stock price is $\$ 17.83$, how much are the options worth? What happens if the stock goes up to \$19.00? 20.00?

## Black-Scholes

$$
\begin{aligned}
& S=17.83, \quad X=20.00, \quad r=1.00, \quad T=2 / 12, \quad \sigma_{2003}=36.1 \% \\
& d_{1}=\frac{\ln (S / X)+\left(r+\sigma^{2} / 2\right) T}{\sigma \sqrt{T}}=-0.694 \\
& d_{2}=d_{1}-\sigma \sqrt{T}=-0.842 \\
& \text { Call price }=S \times N\left(d_{1}\right)-X e^{-r T} N\left(d_{2}\right)=\$ 0.35
\end{aligned}
$$

Cisco stock price, 1993-2003


Cisco returns, 1993-2003


## Cisco option prices



## Option pricing

## Factors affecting option prices

|  | Call option | Put option |
| :--- | :---: | :---: |
| Stock price (S) | + | - |
| Exercise price (X) | - | + |
| Time-to-maturity (T) | + | + |
| Stock volatility $(\sigma)$ | + | + |
| Interest rate $(r)$ | + | - |
| Dividends (D) | - | + |

## Example 2

Call option with $X=\$ 25, r=3 \%$

| Time to expire | Stock price | Std. deviation | Call option |
| :---: | :---: | :---: | :---: |
|  | $\$ 18$ | $30 \%$ | $\$ 0.02$ |
|  | 25 | 30 | 1.58 |
| $\mathrm{~T}=0.25$ | 32 | 30 | 7.26 |
|  | 18 | 50 | 0.25 |
|  | 25 | 50 | 2.57 |
|  | 32 | 50 | 7.75 |
|  | 18 | 30 | 0.14 |
| $\mathrm{~T}=0.50$ | 25 | 30 | 2.29 |
|  | 32 | 30 | 7.68 |
|  | 18 | 50 | 0.76 |
|  | 25 | 50 | 3.67 |
|  | 32 | 50 | 8.68 |

## Option pricing



## Using Black-Scholes

## Applications

> Hedging currency risk
$>$ Pricing convertible debt

## Currency risk

Your company, headquartered in the U.S., supplies auto parts to Jaguar PLC in Britain. You have just signed a contract worth $£ 18.2$ million to deliver parts next year. Payment is certain and occurs at the end of the year.

The $\$ /$ § exchange rate is currently $\mathrm{S}_{\$ / \mathrm{E}}=1.4794$.
How do fluctuations in exchange rates affect \$ revenues? How can you hedge this risk?
$\mathbf{S}_{\text {S/E }}$, Jan 1990 - Sept 2001


## $\$$ revenues as a function of $\mathbf{S}_{\$ / \xi}$



## Currency risk

## Forwards

1 -year forward exchange rate $=1.4513$
Lock in revenues of $18.2 \times 1.4513=\$ 26.4$ million

## Put options*

$$
S=1.4794, \sigma=8.3 \%, T=1, r=-1.8 \% *
$$

| Strike price | Min. revenue | Option price | Total cost $(\times 18.2 \mathrm{M})$ |
| :--- | :--- | :--- | :--- |
| 1.35 | $\$ 24.6 \mathrm{M}$ | $\$ 0.012$ | $\$ 221,859$ |
| 1.40 | $\$ 25.5 \mathrm{M}$ | $\$ 0.026$ | $\$ 470,112$ |
| 1.45 | $\$ 26.4 \mathrm{M}$ | $\$ 0.047$ | $\$ 862,771$ |

*Black-Scholes is only an approximation for currencies; $r=r_{U K}-r_{U S}$

## $\$$ revenues as a function of $\mathbf{s}_{\$ / \neq}$



## Convertible bonds

Your firm is thinking about issuing 10-year convertible bonds. In the past, the firm has issued straight (non-convertible) debt, which currently has a yield of $8.2 \%$.

The new bonds have a face value of $\$ 1,000$ and will be convertible into 20 shares of stocks. How much are the bonds worth if they pay the same interest rate as straight debt?

Today's stock price is $\$ 32$. The firm does not pay dividends, and you estimate that the standard deviation of returns is $35 \%$ annually. Long-term interest rates are 6\%.

## Payoff of convertible bonds



## Convertible bonds

Suppose the bonds have a coupon rate of 8.2\%. How much would they be worth?

Cashflows*

| Year | 1 | 2 | 3 | 4 | $\ldots$ | 10 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Cash | $\$ 82$ | $\$ 82$ | $\$ 82$ | $\$ 82$ |  | $\$ 1,082$ |

Value if straight debt: $\$ 1,000$
Value if convertible debt: $\$ 1,000+$ value of call option

* Annual payments, for simplicity


## Convertible bonds

## Call option

$$
\begin{aligned}
& X=\$ 50, S=\$ 32, \sigma=35 \%, r=6 \%, T=10 \\
& \text { Black-Scholes value }=\$ 10.31
\end{aligned}
$$

## Convertible bond

Option value per bond $=20 \times 10.31=\$ 206.2$
Total bond value $=1,000+206.2=\$ 1,206.2$
Yield $=5.47 \%{ }^{*}$
*Yield $=I R R$ ignoring option value

