Optimization Methods in Management Science

MIT 15.053

Recitation 5

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Problem 1

Suppose we are solving the linear program given in the tableau below:

Basic	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
$(-z)$	2	3	5	2	0	0	0	0
s_1	5	6	-1	2	1	0	0	6
s_2	-3	-1	2	-1	0	1	0	4
s_3	-2	0	2	-2	0	0	1	3

After three pivots, suppose we get the following tableau.

Basic	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
$(-z)$	a	b	c	d	-3	-4	0	e
x_4	1	$3.6\bar{6}$	0	1	$0.6\bar{6}$	$0.3\bar{3}$	0	$5.3\bar{3}$
x_3	-1	$1.3\bar{3}$	1	0	$0.3\bar{3}$	$0.6\bar{6}$	0	$4.6\bar{6}$
s_3	2	$4.6\bar{6}$	0	0	$0.6\bar{6}$	$-0.6\bar{6}$	1	$4.3\bar{3}$

(a) Determine the simplex multipliers, and then compute the optimal value and reduced costs of variables x_1, x_2, x_3, x_4, which are missing in the tableau (i.e., a, b, c, d, e).

(b) Assume that the cost coefficient of variable x_2 is increased by 5 (from 3 to 8). Does the current optimal solution remain optimal?

Problem 2

You are given the following optimization problem:

$$\left.\begin{array}{rrcl} \max & x_1 - 2x_2 & & \\ \text{subject to:} & & & \\ \text{Constr1:} & 3x_1 + x_2 & \geq & 3 \\ \text{Constr2:} & x_1 + 2x_2 & = & 4 \\ & x_1, x_2 & \geq & 0. \end{array}\right\}$$

(a) Put the problem in standard form.

(b) An initial basic feasible solution is not apparent. Therefore, we will apply Phase I to find an initial feasible solution. Write the Phase I problem.

(c) Write the initial tableau for the Phase I problem in canonical form below. The number of additional variables is at most 3. (Note: make sure it is in canonical form and the rhs value of the objective function row is correct!)

Problem 3

Consider the following 2-person zero-sum game:

	C_1	C_2	C_3
R_1	2	3	-2
R_2	3	1	0
R_3	-3	-3	3

(a) Write a linear program to determine an optimal strategy for the row player. Do not solve the linear program.

(b) Write a linear program to determine an optimal strategy for the column player. Do not solve the linear program.

Problem 4

Two players, say Player I and Player II, simultaneously call out one of the numbers one or two. Player Is name is Odd; he wins if the sum of the numbers is odd. Player IIs name is Even; she wins if the sum of the numbers is even. The amount paid to the winner by the loser is always the sum of the numbers in dollars. It turns out that one of the players has a distinct advantage in this game. Can you tell which one it is?