Game Theory for Strategic Advantage

15.025

Alessandro Bonatti

Building a Language: Recap

- Dominated Strategy
  - From my own perspective, can I switch to a single different strategy, and improve my payoff *irrespective* of others' behavior?

- Best response
  - Given my own probabilistic assessment of others' behavior, what's the strategy that yields the highest expected payoff?

- Nash equilibrium
  - With hindsight (after the game), does either player regret choosing her action, holding fixed what other players chose?

Penalty Kicks

- Middle is not dominated for the Kicker.
- Is it ever a best response?

		Goalie: Left	Goalie: Right
Kicker	Left	2 , 5	5 , 1
	Middle	3 , 3	3 , 3
	Right	5 , 1	2 , 5

- Let *x* be the kicker's assessed probability (belief) that the goalie will go Left.

- Compute (and draw) expected payoffs:
  - Shoot Left → ***2x+5(1-x) = 5-3x***
  - Shoot Middle → ***3x+3(1-x) = 3***
  - Shoot Right → ***5x+2(1-x) = 2+3x***

Middle is never a best response!

Chocolate: Split or Steal?

of truffles

		Column	
		Split	Steal
Row	Split	(2, 2)	(0, 4)
	Steal	(4, 0)	(0, 0)

- Seems like a Prisoners' Dilemma

  But wait…

- **Is this the right game?**
  - **payoff matrix**
  - **(available actions)**

Total Utility = Chocolate + … ?

		Column	
		Split	Steal
Row	Split	**(2 +… , 2 +…)**	**(0 +… , 4 +…)**
	Steal	**(4 +… , 0 +…)**	**(0 +… , 0 +…)**

- … fairness
- … reputation
- … continuation play (future payoffs if repeated interaction)
- … premium for winning (value of future monopoly)

Once the blanks are filled, let the game speak!

Then, you may revisit your assumptions

Stag Hunt

(J.-J. Rousseau, 1754)

		Column	
		Stag	Rabbit
Row	Stag	(2, 2)	(-1, 0)
	Rabbit	(0, -1)	(0, 0)

- What if "looking selfish" is worth **-5** chocolates, and
- "being cheated" is worth **-1** ?
- Hunting stag vs. rabbit (*steal = rabbit*)
- No dominated strategies
- Two Nash equilibria

Today's Learning Goal

- Given a game,
- how to predict behavior:
  - focal points
  - tipping points
  - evolutionary dynamics
  - stable frequencies

Dividing the Markets…

- Consider the following list of US cities: *Dallas, New York, Philadelphia, Phoenix, San Francisco, Seattle, St. Louis, and Washington DC.*

- There are two players. Each player (without communicating) must select one or more cities from this list, with the following restrictions, and to maximize the following payoffs.

- **Game 1 (Coordination)**: Player 1 must take New York & player 2 must take San Francisco. Both players win $100 if the list is exactly divided (no overlaps, no omissions); both players get $0 otherwise.

- **Which cities would you pick? Why?**

… Nash Equilibria

- Suppose you are sure your “opponent” is picking (NY, PHL, WAS) → Your **best response**?
- Suppose the cities are **partitioned**
- Does either player want to change her list?
  No → all **partitions** are **Nash Equilibria**
- Ok, now how do you pick a list?

Focal Points

"You are to meet somebody in New York City. You have not been instructed where to meet; you have no prior understanding with the person on where to meet; and you cannot communicate with each other. You are simply told that you will have to guess where to meet and that he is being told the same thing and that you will just have to try to make your guesses coincide."

Schelling (*Strategy of Conflict*, 1960)

Grabbing the Markets

Game 2 (Preferred equilibria): each player gains $20 for each city on her list, but loses $50 for each city on both lists.

- **Good approximation for spectrum auctions**
- Any **partition** of the cities is an equilibrium.
- In this game, **NE ➔ no money left on the table**!
- Players fight over share of the pie
- Miscoordination risk!

Grab the Last City

One city is left on the list.
Leave the city **guarantees zero.**
Both take → -30 each
Only you take → +20

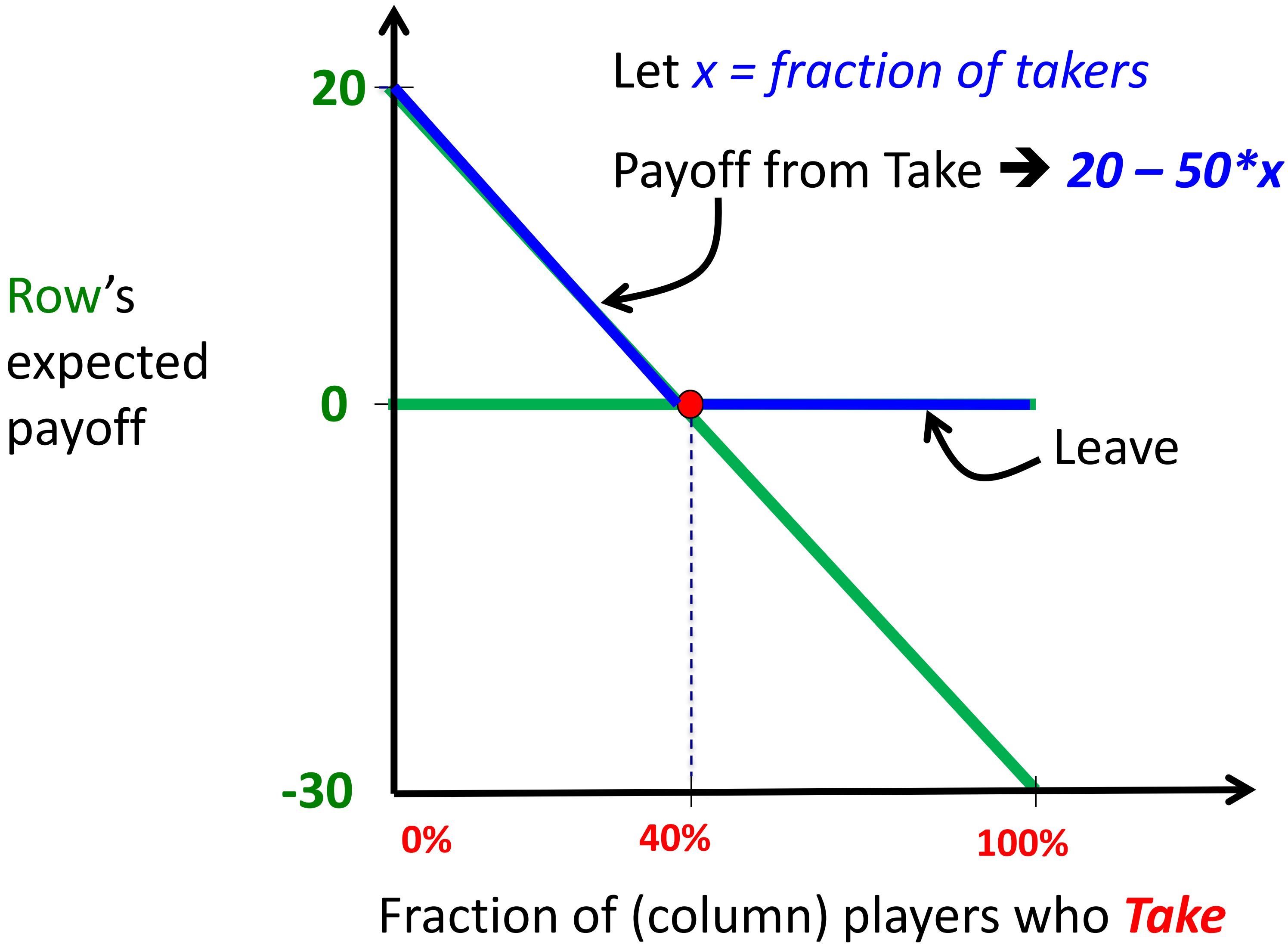
		Column: Take	Column: Leave
Row	**Take**	**(-30, -30)**	**(20, 0)**
	Leave	**(0, 20)**	**(0, 0)**

Rationalizable Outcomes = **all four!!**
Nash Equilibria = (**Take , Leave**) and (**Leave , Take**)

An Experiment

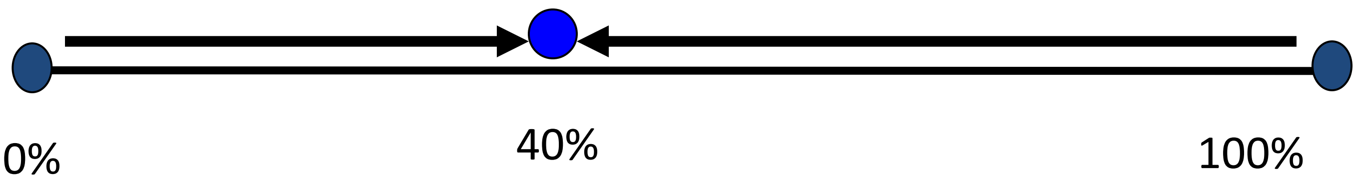
- Go to the link Natallia emailed you
- Select “Take” or “Leave”
- Randomly matched within the class
- Keep track of your profits over time
- Maximize your total profits

Expected Payoffs in "Grab the City"



Evolution in “Grab the City”

Prob. of Take in population



- Row and Col players are drawn from the same population
- Too many *Take* → *Leave* gets higher payoff
- Too many *Leave* → *Take* gets higher payoff

Who Chooses "Take"?

- Stability says your opponent chooses "Take" with 40% probability.
- But in practice, what action will you choose?
  – Expected payoff (take) = leave
- Suppose players are slightly different:
  – Risk aversion
  – Outside options (value of "Leave")
- Minor differences can serve as tie-breakers
- Correlation with earlier answers?

Q4) If you were a manager deciding whether to enter (= grab) a market with those payoffs (*i.e.,* with room for one firm only), what odds would you require in order to “go in”?

Explicit Randomization: Penalty Kicks

ACTUAL PAYOFFS

		Goalie Left	Goalie Right
Kicker	Left	64 , 36	94 , 6
	Right	89 , 11	44 , 56

Stable fractions: Kicker chooses left **60%** of the time
Goalie chooses left **67%** of the time

Several European Football Leagues

Actual fractions: Kicker chooses left **54%** of the time
Goalie chooses left **58%** of the time

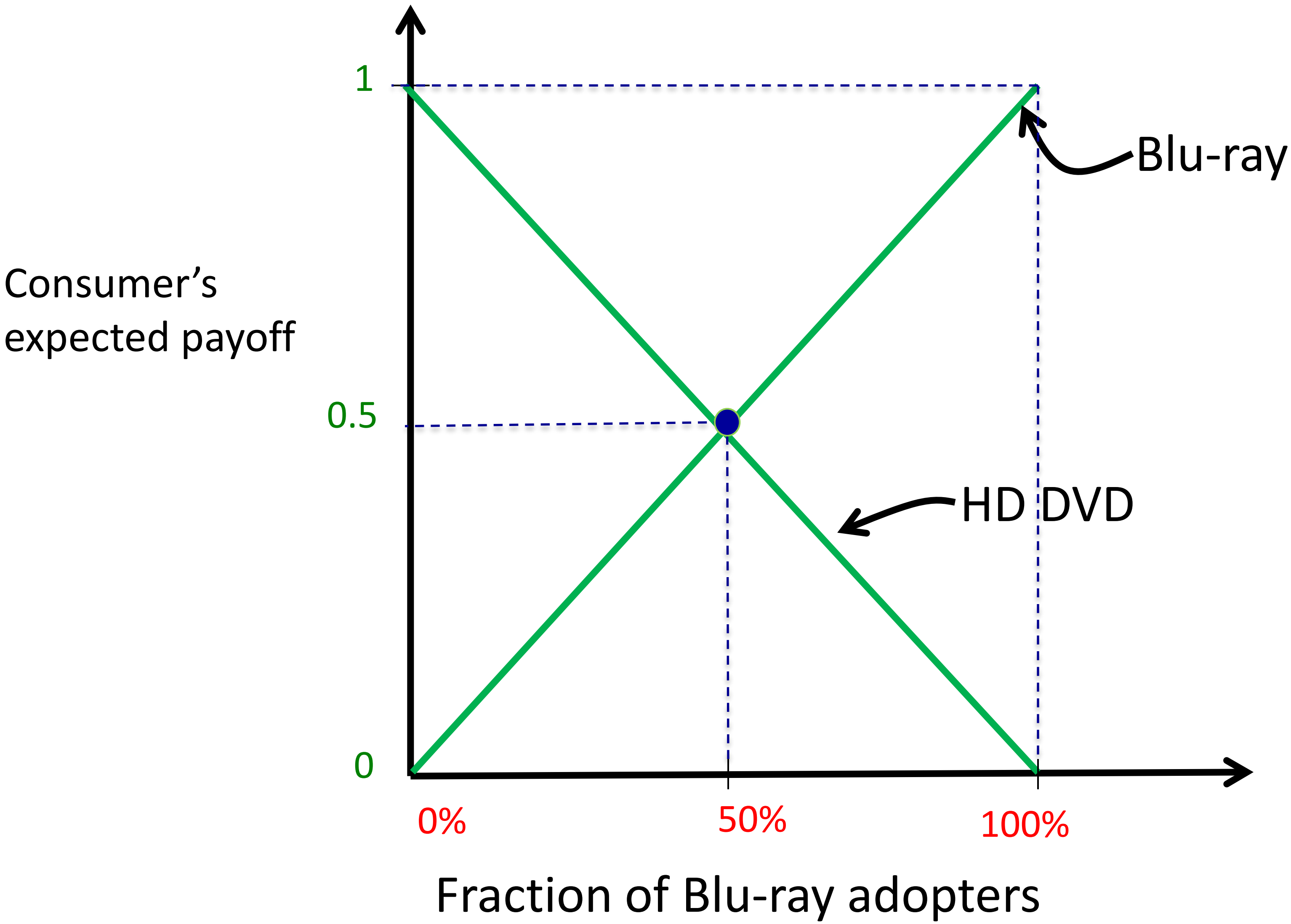
Cannot statistically reject "No Difference"

Technology Adoption

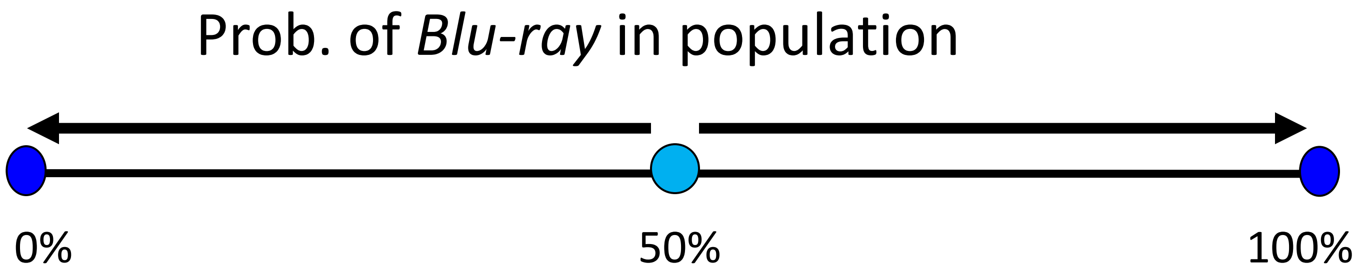
		Column	
		Blu-ray	**HD DVD**
Row	**Blu-ray**	**(1, 1)**	**(0, 0)**
	HD DVD	**(0, 0)**	**(1, 1)**

- The role of consumer coordination
- Positive spillovers
- Indirect and direct network externalities

Expected Payoffs in Technology Adoption



Evolution in Technology Adoption



- Row and Col players are drawn from the same population
- A 50:50 split of the population is unstable – why?

- What happened?
- Role of large players (Sony, Warner, Walmart)

Takeaways: How to anticipate the behavior of others

- **Rationality**: they will never play a dominated strategy
- **Rationalizability**: they will play a best response to some beliefs about what others play
- **Equilibrium**: they will play a best response to the *correct* beliefs about others
- **Evolution**: they will play in an adaptive way (e.g., observe past play, reoptimize)

Next class: how to *steer* others' behavior!

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Spring 2015