# 17.881/882 Problem Set 5

1. Colonel Blotto has 3 divisions with which to defend 2 mountain passes. His opponent also has 3 divisions. Blotto successfully defends a pass if and only if he allocates an equal or greater number of divisions to it as his opponent. Blotto wins if he successfully defends *both* passes, and loses otherwise. Blotto's payoff is 1 if he wins, -1 if he loses; the opponent's payoff is 1 if he wins, and -1 if he loses.

a)Find a mixed-strategy Nash Equilibrium for the game.

b)If it is unique, prove that it is so. If not, find another Nash Equilibrium.

2. A set of $n$ firms seek to bribe the government into granting them a monopoly. The monopoly is worth $W_i$ to each firm $i = 1, ...n$, where $W_i = W$ for all i. Firm i offers bribes of $x_i$. Given the bribe vector $(x_1, ..., x_n)$, the probability the government grants the monopoly to firm $i$ is

$$p_i(x_1, ..., x_n) = \frac{\alpha_i x_i^r}{\sum_{j=1}^n \alpha_j x_j^r}$$

Firm $i$'s expected payoff is

$$\pi_i(x_1, ..., x_n) = p_i(x_1, ..., x_n)W - x_i$$

In solving the model, make any assumption you need on $W$.

(a) Suppose $n = 3$, $\alpha_1 = \alpha_2 = 2$, $\alpha_3 = 1$, and $r = 1$. Find a pure-strategy Nash equilibrium.

(b) Suppose $n = 3$, $\alpha_i = 1$ for all $i$, and $r = 2$. Find a pure-strategy Nash equilibrium.