1.2 Building galaxies

As we saw above, gravitational scattering plays a very small role in the dynamics of stellar orbits within galaxies or globular clusters. We return to our original question, "what is a galaxy?" In light of the previous section, it seems better to think of a galaxy as a collisionless fluid of stars, a superposition of a large number of orbits guided by a single background potential.

How does one put together such a galaxy? Superpose orbits! Schwarzschild's method (see Figure 1.2) follows these simple steps:

- choose gravitational potential

- generate an orbit library

- populate potential with orbits
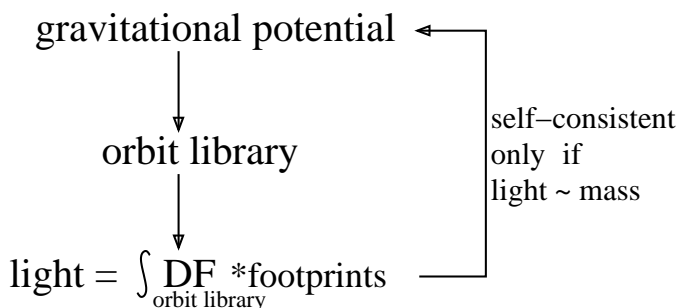
- check for self-consistency

Figure 1.2: Schwarzschild's method for building galaxies. One adjusts the distribution function (DF) to match the light distribution or the gravitational potential.

For example, imagine building a spherically symmetric galaxy. How would the orbits be distributed in phase space? Even though they may occupy three-dimensional space quite uniformly, there is no reason to insist that the orbits should be uniformly distributed in momentum space. A galaxy made only of circular orbits would appear "hot" in the tangential direction and "cold" in the radial direction, while a collection of radial orbits (an equally valid approach to a spherical system) would look hot in the radial direction and cold in the tangential direction (see Figure 1.3).
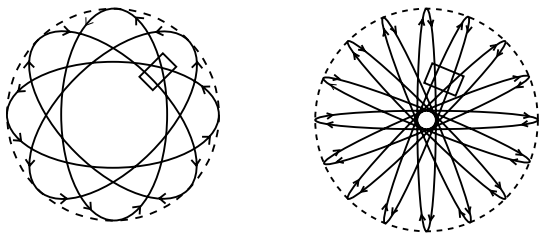

Figure 1.3: Two schemes for constructing a spherical galaxy: out of circular orbits, left, and out of (nearly) radial orbits, right.

"Hot" and "cold" can be understood quantitatively by defining a symmetric *velocity dispersion* tensor:

$$\sigma_{ij}^2 = \overline{(v_i - \overline{v}_i)(v_j - \overline{v}_j)}, \tag{1.14}$$

where $\overline{v}_i$ is the velocity in the $\hat{\mathbf{e}}_i$ direction averaged at some point. For a collisionless fluid, we can diagonalize the velocity dispersion tensor, giving three independent temperatures at each point in
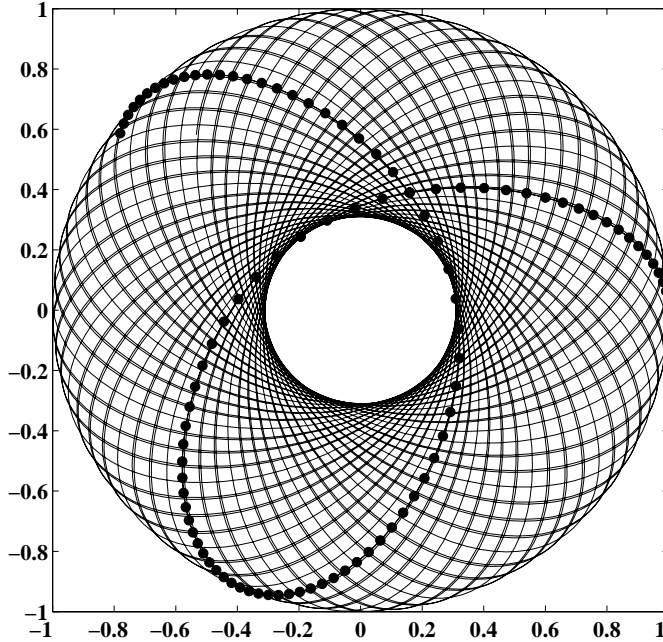
Figure 1.4: The orbit of a star on a spherically symmetric logarithmic potential. The position of the star is plotted for the first 100 time steps. By following the star for many orbits we accumulate a probability density "footprint" that may be used to generate a galaxy model.
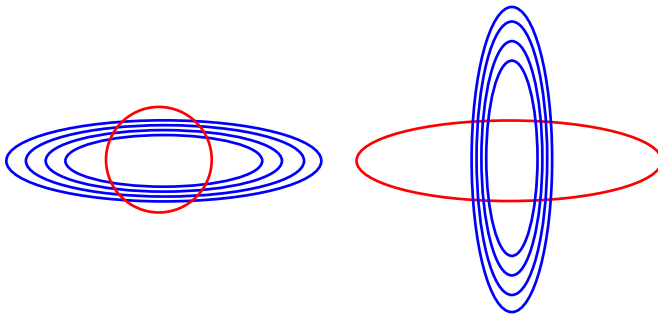

Figure 1.5: Left: a disk in circular orbit around a spherical bulge. Right: a disk in polar orbit around a triaxial bulge. Such "polar ring" galaxies constitute perhaps 0.1% of all galaxies.

space. If the random velocities are greater than (or in the same order as) the ordered velocities, the fluid is *hot*; if the ordered velocities are much greater, the fluid is *cold*.

Observations (Figure 1.7) show that many galaxies have nearly constant rotational velocities with $v(r) \approx v_c$ over the observable range of radius. These flat rotation curves are a departure from Keplerian orbital velocities, which would scale as $v(r) \sim r^{-1/2}$. Flat rotation curves imply mass increasing linearly with radius, and so cannot continue without bound.

For a spherically symmetric system, the centripetal acceleration is given by

$$\frac{v^2}{r} = \frac{d\Phi}{dr} \tag{1.15}$$

For a constant rotation velocity $v(r) = v_c$, the solution for the potential is

$$\Phi = v_c^2 \ln \frac{r}{r_{\text{ref}}}. \tag{1.16}$$

Combining with Poisson's equation $\nabla^2 \Phi = 4\pi G \rho$, we find an expression for the density profile:

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}. \tag{1.17}$$

Telescope

Focal Plane

Collimator

Grism   Spectrometer
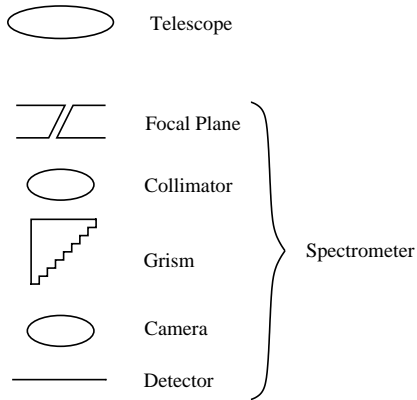
Camera

Detector

Figure 1.6: A schematic representation of a spectrograph. Light from the telescope is brought to a focus in the focal plane. The slit of the spectrograph isolates the light from a narrow strip of the sky, which diverges and passes through the collimator. Parallel light rays from the collimator enter the grism (a grating ruled on a prism) which disperses the different wavelengths. The camera brings the light to a focus on the detector.

galaxy

spectrograph slit

[N II]
H α
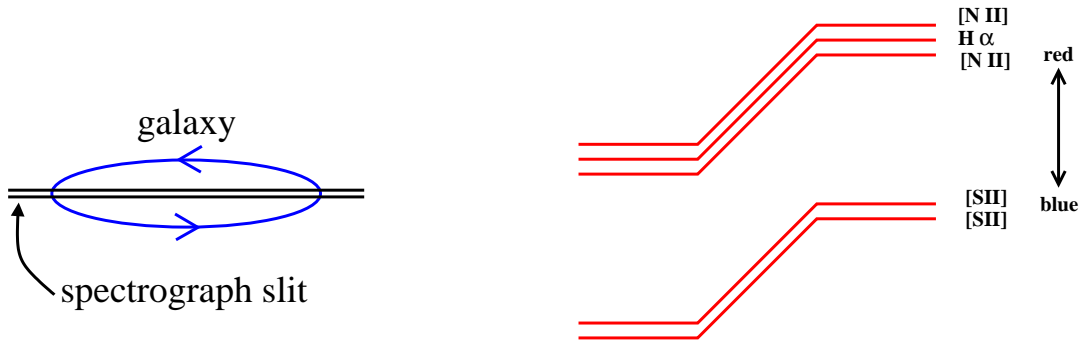[N II]   red

[SII]   blue
[SII]

Figure 1.7: Left: The slit of a spectrometer is placed along the major axis of a disk galaxy. The disk is highly inclined to the line of sight. Stars and gas on the left side of the slit have a line of sight velocity toward us (relative to the center of the galaxy). Right: The spectrum of the galaxy. The lines shown are typical of gas photo-ionized by hot stars. The lines are blueshifted on the left side and redshifted on the right side. This gives the *rotation curve* of the galaxy. The rotation curve is flat in the outer parts of the galaxy and roughly linear in the inner parts.
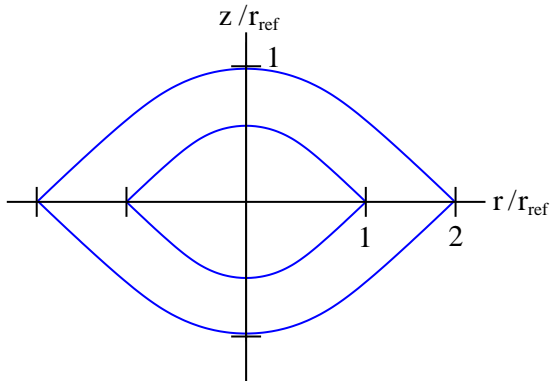
Figure 1.8: Equipotentials for Mestel's Disk

Among the obvious shortcomings of this model are the failure of the potential to go to zero at large radius and the prediction of infinite mass. Notwithstanding, it is still a useful potential for many practical applications.

Another weakness of this model is that it is limited to spherically symmetric galaxies and we know from experience that many galaxies are shaped more like flattened disks. A corresponding model for such a galaxy is called a *Mestel disk*, which is azimuthally symmetric around the z-axis. The potential for the Mestel disk is

$$\Phi(r,\theta) = v_c^2 \left[\ln \frac{r}{r_{\text{ref}}} + \ln \frac{1 + |\cos\theta|}{2}\right], \tag{1.18}$$

where θ is measured from the positive z-axis. The Laplacian in spherical geometry with azimuthal symmetry is given by

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}. \tag{1.19}$$

Applying Poisson's equation to equation (1.18), we find that the Mestel disk has zero density everywhere except $\theta = \pi/2$, where the density is infinite. The more useful (and in this case finite) quantity is the *surface density*, determined by integrating the density over the (infinitesimal) thickness of the disk:

$$\Sigma(r) = \int \rho(r,z)dz = \frac{v_c^2}{2\pi G r} \quad . \tag{1.20}$$

Like the spherical model for a flat rotation curve, the disk model also suffers from an infinite total mass and radius, but it is still useful for many applications.

Observations show that most galaxies have elements of *both* of the above models, with a roughly spherical bulge *and* a flattened disk. The relative weight of these features (the "disk-to-bulge ratio," Figure 1.9) is a useful parameter for classifying galaxies, with disk strength correlating positively with other galaxy properties such as young stellar fraction, hydrogen fraction, CO gas, dust, and the strength of spiral arms. In most galaxies the spiral arms, which are located within the disk are the primary location for new star formation. The disk and bulge components of most galaxies appear to have had rather different histories, a subject to be raised again in the context of galaxy formation.

Soon after the realization that many nebulae were in fact distinct galaxies, astronomers (and in particular Hubble) began systematically classifying galaxies in the hope that taxonomy would lead to further understanding. The principal component of Hubble's classification scheme is strongly correlated with disk-to-bulge ratio. But Hubble conflated the relative proportions of bulge and disk

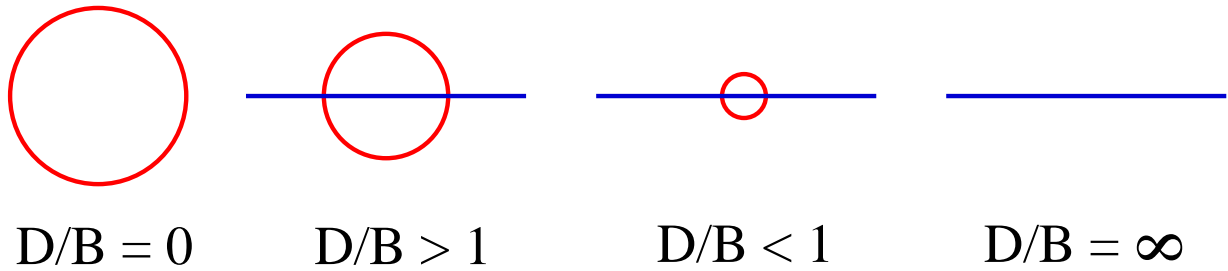$$D/B = 0 \qquad D/B > 1 \qquad D/B < 1 \qquad D/B = \infty$$

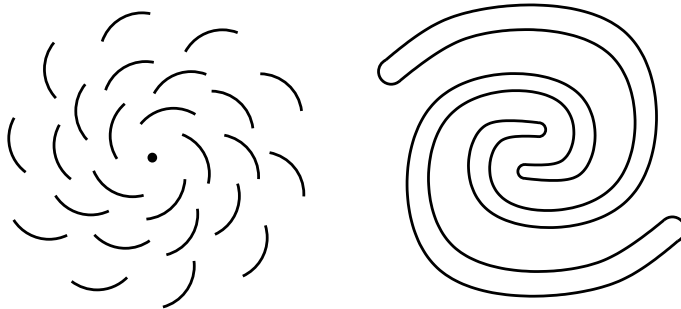Figure 1.9: Galaxies with varying disk to bulge ratios, seen edge on.

Figure 1.10: Two spiral galaxies, *flocculent* to the left and *grand design* to the right. Flocculent systems are more likely to be classified Sa; grand designs are more frequently Sc.

with the prominence of spiral arms within the disk. In Hubble's scheme most galaxies were classified as "E", "Sa", "Sb" or "Sc." "E" stood for elliptical with no sign of spiral structure. "S" stood for spiral structure, with "Sa" galaxies showing the least spiral structure and "Sc" galaxies showing the most, as shown in Figure 1.10. The sequence from E to Sc is approximately one of increasing disk-to-bulge ratio. But there are also galaxies with prominent disks and little evidence for spiral structure. Hubble added the S0 classification, placing it between E and Sa. Modern versions of Hubble's scheme have put the S0 systems parallel to the S systems, with S0/a, S0/b and S0/c systems having increasingly large disk-to-bulge ratios. One might hope that disk-to-bulge ratio would completely replace Hubble's system, but while easy to describe, disk-to-bulge ratio can be difficult to measure, particularly for systems that are face-on. Modern "quantitative" classification schemes often use central concentration as a proxy for disk-to-bulge ratio and use deviation from bilateral symmetry as a proxy for spiral structure.