**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**Physics 8.901: Astrophysics I** **Spring Term 2006**

**PROBLEM SET 3**

**Due:** Thursday, March 9 in class

**Reading:** Chapter 4 in Hansen, Kawaler, & Trimble (including §4.8). You should also at least look through Chapter 5 on convection.

1. **Radiation pressure and the Eddington limit.**

   (a) Show that the condition that an optically thin cloud of material can be ejected by radiation pressure from a nearby luminous object is that the mass to luminosity ratio ($M/L$) for the object be less than $\kappa/(4\pi Gc)$, where $\kappa$ is the mass absorption coefficient (assumed to be independent of frequency). (*Hint:* The force per unit mass due to radiation pressure on absorbing material is $\int(\kappa_\nu F_\nu/c)d\nu$, where $F_\nu$ is the radiative flux per unit frequency.)

   (b) Calculate the terminal velocity $v$ attained by such a cloud under radiation and gravitational forces alon, if it starts from rest at a distance $R$ from the object. Show that

   $$v^2 = \frac{2GM}{R}\left(\frac{\kappa L}{4\pi GMc} - 1\right).$$

   (c) A minimum value for $\kappa$ may be estimated for pure hydrogen as that due to Thomson scattering off free electrons, when the hydrogen is completely ionized. The Thomson cross-section is $\sigma_{\rm T} = 6.65 \times 10^{-25}$ cm$^2$. The mass scattering coefficient is thus $> \sigma_{\rm T}/m_{\rm H}$, where $m_{\rm H}$ is the mass of a hydrogen atom. Show that the maximum luminosity that a central mass $M$ can have and still not spontaneously eject hydrogen by radiation pressure is

   $$L_{\rm Edd} = \frac{4\pi cGMm_{\rm H}}{\sigma_{\rm T}} = 1.3 \times 10^{38}(M/M_\odot)\ {\rm erg\ s^{-1}}.$$

   This is called the *Eddington limit*.

2. **Stability against convection.**

   (a) In lecture, we derived the following condition for stability against convection in a star,

   $$\frac{\rho}{\gamma P}\frac{dP}{dr} - \frac{d\rho}{dr} > 0,$$

   where $P$ is the pressure, $\rho$ is the density, $\gamma$ is the exponent in the adiabatic equation of state ($P = K\rho^\gamma$), and $r$ is distance from the stellar center. Using the ideal gas law $P = \rho kT/\mu m_p$, show that this condition can also be written as

   $$\frac{dT}{dr} > \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}.$$

   Furthermore, use the relevant stellar structure equations (for a radiative star) to show that this reduces to

   $$L(r) < \left(1 - \frac{1}{\gamma}\right)\frac{16\pi acT^4GM(r)}{3\kappa P},$$

   where $L(r)$ and $M(r)$ are the luminosity and enclosed mass at radius $r$, $\kappa$ is the opacity, and $a$ is the radiation constant.

(b) Show that to avoid convection in a stellar region where the equation of state is that of an ideal monatomic gas, the luminosity at a given radius must be limited by (in cgs units)

$$L(r) < 1.22 \times 10^{-18} \frac{\mu T^3}{\kappa \rho} M(r)$$

where $T(r)$ is the temperature, $\mu$ is the mean molecular weight, $\kappa$ is the Rosseland mean opacity, and $M(r)$ is the mass enclosed at a radius $r$.

3. **Radiative transfer.** HK&T, Problem 4.1.

4. **Helium ionization.** HK&T, Problem 4.6. (Note that this problem refers back to HK&T Problem 3.1, which is essentially Problem 3 from our Problem Set 2.)

5. **Stimulated emission.** HK&T, Problem 4.9.