8.851 Homework 8

Iain Stewart, April 16, 2003 (due Apr.23)

Problem 1) Heavy Meson Chiral Perturbation Theory

Show that we recover our results for the leading order heavy meson chiral Lagrangian gauged under electromagnetism by first using left and right hand currents and then specializing to the $U(1)$. Explain how the Q' term must be treated.

Problem 2) Do Problem 3 in Chapter 5 of the Book

Problem 3) Chiral Lagrangian for Heavy Vector Mesons

Introduce the vector meson fields as a 3×3 octet matrix

$$\mathcal{O}_\mu = \begin{bmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\phi_\mu^{(8)}}{\sqrt{6}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\phi_\mu^{(8)}}{\sqrt{6}} & K_\mu^{*0} \\ K_\mu^{*-} & \overline{K}_\mu^{*0} & -\frac{2\phi_\mu^{(8)}}{\sqrt{6}} \end{bmatrix}, \tag{1}$$

and as a singlet

$$S_\mu = \phi_\mu^{(0)}. \tag{2}$$

(a linear combination of $\phi_\mu^{(0,8)}$ will be the ϕ and ω). Under chiral $SU(3)_L \times SU(3)_R$ let

$$\mathcal{O}_\mu \to U \mathcal{O}_\mu U^\dagger, \qquad S_\mu \to S_\mu. \tag{3}$$

Under charge conjugation,

$$C\mathcal{O}_\mu C^{-1} = -\mathcal{O}_\mu^T, \; CS_\mu C^{-1} = -S_\mu, \; C\xi C^{-1} = \xi^T. \tag{4}$$

Construct the $\mathcal{O}(p)$ chiral $SU(3)$ Lagrangian for these vector mesons treating them as heavy static fields with fixed four-velocity v^μ. Comment on the expansion parameter in this theory. (Bonus: Discuss the terms with the quark mass matrix as well.)