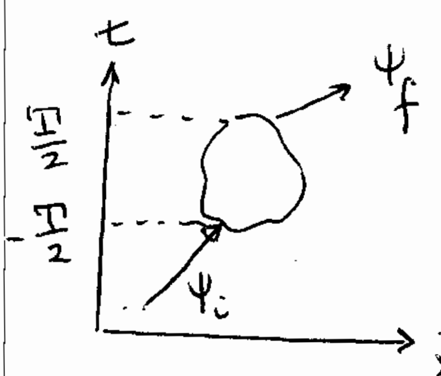


Pertubative transition calculation

Let $H_0 \phi_n = E_n \phi_n$ with $\int \phi_m^* \phi_n d^3x = \delta_{mn}$

Solve $H\psi = (H_0 + V)\psi = i \frac{\partial \psi}{\partial t}$; $\psi = ?$

Expand $\psi = \sum_n a_n(t) \phi_n(\vec{x}) e^{-iE_n t}$

$\therefore \sum_n V a_n \phi_n(\vec{x}) e^{-iE_i t} = i \sum_m \frac{da_n}{dt} \phi_m(\vec{x}) e^{-iE_f t}$

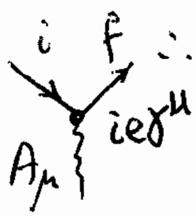
multiply ϕ_f^* and integrate d^3x :

Thus $\frac{da_f}{dt} = -i \sum_n a_n(t) \int \phi_f^* V \phi_n \, d^3x \, e^{i(E_f - E_i)t}$

Assume $a_i(-T/2) = 1$; $a_{n \neq i}(-T/2) = 0$, i.e. only ith state

Thus $\frac{da_f}{dt} = -i \int d^3x \, \phi_f^* V \phi_i \, e^{i(E_f - E_i)t}$ at $t = -\frac{T}{2}$,

Assume V and the transition are small, the initial conditions are valid for $\frac{-T}{2} \leq t \leq T/2$



$\therefore \quad a_f(\frac{T}{2}) = -i \int d^4x \, \phi_f^* V \phi_i \, e^{i(E_f - E_i)t}$

$= -i \int d^4x \, \psi_f^* V \psi_i \equiv T_{fi}$ = transition amp.$_{i \to f}$

for $\gamma^0 V = -e\gamma^\mu A_\mu$, i.e. e^- interacts with EM field A_μ.

$$T_{fi} = ie \int \bar{\psi}_f \gamma^\mu A_\mu \psi_i \, d^4x = -i \int (\underbrace{-e \bar{\psi}_f \gamma^\mu \psi_i}_{= \, j^\mu \text{(current)}}) A_\mu \, d^4x$$

$\psi_i = u_i e^{-iP_i x}$ $\quad \bar{\psi}_f = \bar{u}_f e^{+iP_f x}$

$$T_{fi} = -ie \overbrace{(\bar{u}_f \gamma^\mu u_i) A_\mu}^{M = \text{Amplitude}} (2\pi)^4 \delta^4(P_f^4 - P_i^4) \; ; \quad A_\mu = \sum_i \epsilon_\mu^i \cdot a_i$$

creation ↑ ↑ Annihlation operators

ϵ_μ^i : polarization

Crosssections $P_1 + P_2 \to q_1 + q_2 + \cdots + q_n$

decay : $P \to q_1 + q_2 + \cdots + q_n$

$$d\sigma = \frac{1}{2s} \prod_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3 q_i}{2E_i} (2\pi)^4 \delta^4(P_1 + P_2 - \sum_{i=1}^{n} q_i) \langle |M|^2 \rangle$$

$$d\Gamma = \frac{1}{2M} \prod_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3 q_i}{2E_i} (2\pi)^4 \delta^4(P - \sum_{i=1}^{n} q_i) \langle |M|^2 \rangle$$

Define $\delta^4(P - \sum q_i) \prod_{i=1}^{n} \frac{d^3 q_i}{2E_i} = d(PS)_n$

for $m_1 = m_2 = \cdots = m_n = 0$, we have

$$d(PS)_2 = \frac{1}{8} d\Omega = \frac{\pi}{4} d\cos\theta = \frac{\pi}{2}$$

$$d(PS)_3 = \frac{1}{8} dE_1\, dE_2\, d\Omega_1\, d\varphi_{12} = \pi^2 dE_1\, dE_2$$

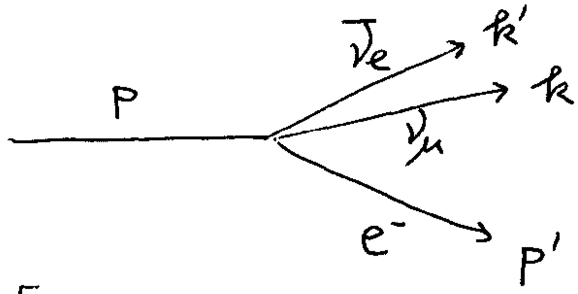
$$= \pi^2 E_1\, dE_1$$

(integrate dE_2 from $\frac{M}{2} - E_1$ to $\frac{M}{2}$)

(integrate dE_1 from 0 to $\frac{M}{2}$) $= \pi^2 M^2 / 8$

Determine G from μ^- decay

$$\mu^-(p) \to e^-(p') + \bar{\nu}_e(k') + \nu_\mu(k)$$ see 12.5 in Q&L



$m_e \sim m_\nu \sim 0$

$$M = \frac{G}{\sqrt{2}}\left[\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)\right]\left[\bar{u}(p')\gamma_\mu(1-\gamma^5)v(k')\right]$$

$$\sum_{\text{spins}} M^\dagger M = \frac{G^2}{2}\left[\underbrace{\bar{u}(p)}_{\sum_s = \not{p}}(1-\gamma^5)\gamma^\mu \underbrace{u(k)\right]\left[\bar{u}(k)}_{\sum_s = \not{k}}\gamma^\nu(1-\gamma^5)u(p)\right]$$

$$\left[\underbrace{\bar{v}(k')}_{\sum_s = \not{k}'}(1-\gamma^5)\gamma_\mu \underbrace{u(p')\right]\left[\bar{u}(p')}_{\sum_s = \not{p}'}\gamma_{\cancel{\mu}\nu}(1-\gamma^5)v(k')\right]$$

$$= \frac{G^2}{2}\mathrm{Tr}\left[(1-\gamma^5)\ \gamma^\mu\ \not{k}\ \gamma^\nu\ (1-\gamma^5)\ \not{p}\ \right]$$

$$\left[(1-\gamma^5)\ \gamma_\mu\ \not{p}'\ \gamma_\nu\ (1-\gamma^5)\ \not{k}'\ \right] \quad \text{use (12.29)}$$

In μ's rest frame

$$= 128\ (p_\mu \cdot k'^\mu)(p'_\nu \cdot k^\nu) \qquad p'_\nu = m_\mu - k_\nu - k'_\nu$$

$$= 128\, G^2 (m_\mu k'_0)(m_\mu^2 - 2m_\mu k'_0)\tfrac{1}{2} \qquad 2p'_\nu \cdot k^\nu = (p'_\nu + k)^2$$

$$= (p - k')^2$$

$$d\Gamma = \frac{1}{2m_\mu}\frac{1}{(2\pi)^5}\frac{1}{2}\sum_s M^\dagger M\ \frac{d^3p'}{2E'}\frac{d^3k'}{2k'_0}\frac{d^3k}{2k_0}\delta^4(p - p' - k' - k)$$

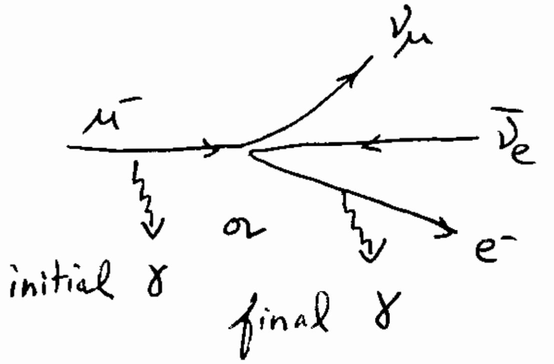
$$\Gamma = \frac{m_\mu G^2}{\pi^3}\int_0^{m_\mu/2} dk'_0\ (k'_0)^2\left(\frac{m_\mu}{2} - k'_0\right) = \frac{G^2 m_\mu^5}{192\pi^3}$$

Use $m_\mu = 0.106$ GeV

P.39 $\Gamma_\mu = 2.996 \times 10^{-19}$ GeV $\Big\} \Rightarrow G = 1.16 \times 10^{-5}\ \mathrm{GeV}^{-2}$

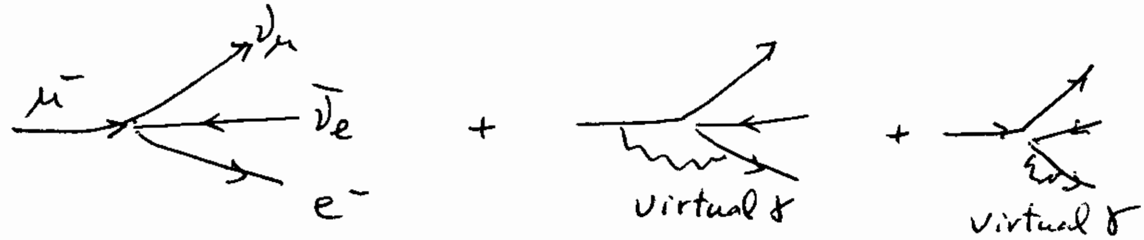
How to compare $\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G^2 m_\mu^5}{192\pi^3}$ with Data?

There are radiative corrections: charges emit γ when accelerated!
($\because \vec{v}$ of $\mu^- \neq \vec{v}$ of e^-)



$= (M_1)^2 \propto \alpha$

$$\delta_{soft\,\gamma} = \frac{2\alpha}{\pi}\left\{\left(\ln\frac{s}{m_e^2} - 1\right)\ln\frac{2k^{max}}{m_\gamma} + \cdots\right\} \xrightarrow[\text{as } m_\gamma \to 0]{} \infty$$



$M_0 \quad + \quad M_V$

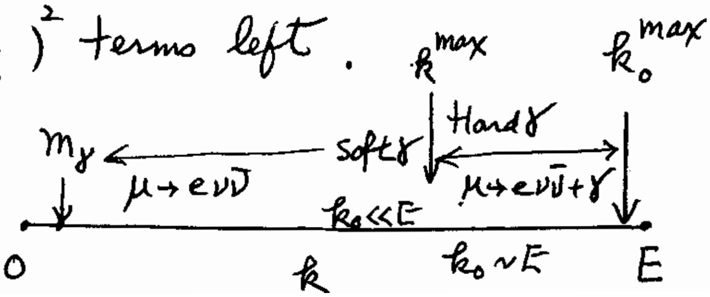
$$\delta_V = 2M_V^* M_0 = \frac{2\alpha}{\pi}\left\{\ln\left(\frac{s}{m_e^2} - 1\right)\ln\frac{m_\gamma}{2E} + \cdots\right\} \xrightarrow[\text{as } m_\gamma \to 0]{} -\infty$$

But

$$\delta_{soft} + \delta_V = \frac{2\alpha}{\pi}\left\{\left(\ln\frac{s}{m_e^2} - 1\right)\ln\frac{k^{max}}{E} + \frac{3}{4}\ln\frac{s}{m_e^2} + \frac{\pi^2}{6} - 1\right\}$$

$m_e \ll E$

note $\begin{cases} \text{IR infinitees have cancelled!!} \\ \text{No } (\ln s/m_e^2)^2 \text{ terms left.} \end{cases}$



Experimentally when $k > k^{max}$ hard γ
we can detect γ_k.
If $k < k^{max}$, we can't!