

## 8.513 Problem Set 3

(Dated: September 29, 2004)

### Due Oct 5

1. Consider a particle of mass  $m$  moving in a potential  $V(x) = kx^2 + ux^4$  with  $u > 0$ .
  - (a) Rewrite the Hamiltonian in terms of creation and annihilation operators.
  - (b) Write the quantum partition function as a coherent state path integral.
  - (c) Show directly the equivalence with the imaginary time Feynman path integral over the paths of the coordinate  $x$ . (First change variables in the coherent state path integral to  $x$  and  $p$ , then do the Gaussian integral over  $p$ ).
  - (d) Specialize to  $u = 0$  so that the potential is that of a simple harmonic oscillator. Consider the average energy at finite temperature. Calculate this directly from the coherent state path integral (you must be able to reproduce the well-known result). It is best to work with the discrete time integral and carefully take the continuum limit at the end of the calculation.
2. Consider the Hamiltonian describing small harmonic fluctuations about the mean field ground state of the Bose liquid.

$$H = \sum_k t_k a_k^\dagger a_k + \frac{\Delta_k}{2} (a_k a_{-k} + h.c) \quad (1)$$

in the notation used in class. Diagonalize to obtain the eigenfrequencies of the excitations as a function of  $k$ . What happens to the excitation frequency as  $k \rightarrow 0$ ?