8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids

Problem Set 7

Due April 13, 2004

1. Show that within the Heitler-London approximation for two hydrogen-like atoms located at R_a and R_b, the singlet and triplet variational energies are given by

$$E_{s,t} = E_a + E_b + \frac{V \pm I}{I \pm l^2}$$

where $l = \int d\boldsymbol{r}\phi_a^*(\boldsymbol{r})\phi_b^*(\boldsymbol{r})$ is the overlap integral,

$$V = \int d\boldsymbol{r}_1, d\boldsymbol{r}_2 |\phi_a(\boldsymbol{r}_1)\phi_b(\boldsymbol{r}_2)|^2 (\Delta H)$$

and I is the exchange integral

$$I = \int d\boldsymbol{r}, d\boldsymbol{r}_2, \phi_a^*(\boldsymbol{r}_1)\phi_b^*(\boldsymbol{r}_2)\phi_b(\boldsymbol{r}_1)\phi_a(\boldsymbol{r}_2)(\Delta H)$$

where

$$\Delta H = \frac{e^2}{R_{ab}} + \frac{e^2}{r_{12}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2a}} \ .$$

2. Problem 5, p.723 from Ashcroft and Mermin

5. *Anisotropic Heisenberg Model*

Consider the anisotropic Heisenberg spin Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{\boldsymbol{RR}'} [J_z(\boldsymbol{R}-\boldsymbol{R}')\boldsymbol{S}_z(\boldsymbol{R})\boldsymbol{S}_z(\boldsymbol{R}') + J(\boldsymbol{R}-\boldsymbol{R}')\boldsymbol{S}_\perp(\boldsymbol{R})\cdot\boldsymbol{S}_\perp(\boldsymbol{R}')] \tag{33.71}$$

with $J_z(\boldsymbol{R}-\boldsymbol{R}') > J(\boldsymbol{R}-\boldsymbol{R}') > 0$.

(a) Show that the ground state (33.5) and one-spin-wave states (33.23) remain eigenstates of $\mathcal{H}$, but that the spin wave excitation energies are raised by

$$S\sum_{R}\left[J_z(\boldsymbol{R})-J_z(\boldsymbol{R})\right] \quad . \tag{33.72}$$

(b) Show that the low-temperature spontaneous magnetization now deviates from saturation only exponentially in $-1/T$.

(c) Show that the argument on page 708, that there can be no spontaneous magnetization in two dimensions, no longer works.