Problem 1.

a) $\left(\nabla^2 - \frac{\varepsilon}{c^2}\omega^2\right)\vec{E},\vec{B} = 0 \qquad \nabla_\perp^2(E,B) = 0$

$$\frac{\sqrt{\varepsilon}\,\omega}{c} = k \quad \Rightarrow \quad \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon}}.$$

$$\vec{E} = A\frac{\hat{r}}{r}e^{-i\omega t+kz} \; ; \; \vec{B} = \sqrt{\varepsilon}\,\hat{k}_z\times\vec{E} = \sqrt{\varepsilon}\frac{A\hat{\varphi}}{r}e^{-i\omega t+kz}$$

b) $\vec{E}_{in} = A\frac{e^{-i\omega t+ikz}}{r}\hat{r}$, $\vec{E}_{ref} = \eta A\frac{e^{-i\omega t-ikz}}{r}\hat{r}$.

(end point $z=0$ and $\eta = \frac{E_{ref}}{E_{in}}$)

$\Rightarrow \vec{B}_{in} = \sqrt{\varepsilon}A\frac{e^{-i\omega t+ikz}}{r}\hat{\varphi}$; $\vec{B}_{ref} = -\eta\sqrt{\varepsilon}A\frac{e^{-i\omega t-ikz}}{r}\hat{\varphi}$.

at $z=0$. $\begin{cases} \vec{E} = (1+\eta)A\frac{e^{-i\omega t}}{r}\hat{r}\,; \\ \vec{B} = (1-\eta)\sqrt{\varepsilon}A\frac{e^{-i\omega t}}{r}\hat{\varphi}\,; \end{cases}$

$$\frac{4\pi}{c}I = \oint\vec{B}\cdot d\vec{l} = (2\pi)(1-\eta)\sqrt{\varepsilon}A\,e^{-i\omega t}$$

$$V = \int_a^b E\,dr = (1+\eta)A\ln\frac{b}{a}\cdot e^{-i\omega t}$$

$$\Rightarrow R = \frac{V}{I} = \frac{(1+\eta)\ln\frac{b}{a}\cdot 2}{(1-\eta)\sqrt{\varepsilon}\cdot c} = \frac{1+\eta}{1-\eta}\cdot\frac{2\ln\frac{b}{a}}{\sqrt{\varepsilon}c}.$$

$$\Rightarrow \eta = \frac{\sqrt{\varepsilon}RC - 2\ln b/a}{\sqrt{\varepsilon}RC + 2\ln b/a} = \frac{E_r}{E_i}.$$

c)

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}(1-\eta^2)\sqrt{\varepsilon}A^2\frac{\hat{z}}{r^2}\cos^2\omega t.$$

$$\Rightarrow \quad P_{wave} = \int_a^b \vec{S}\cdot d\vec{s} = \int |\vec{S}|\cdot 2\pi r\,dr$$

$$= \frac{c}{2}(1-\eta^2)\sqrt{\varepsilon}A^2 \ln\frac{b}{a}\cdot\cos^2\omega t.$$

$$P_{resist} = I\cdot V = \frac{c}{2}(1-\eta)\sqrt{\varepsilon}A\cos\omega t\cdot(1+\eta)A\ln\frac{b}{a}\cos\omega t$$

$$= \frac{c}{2}(1-\eta^2)\sqrt{\varepsilon}A^2\ln\frac{b}{a}\cos^2\omega t.$$

Problem #2 EM waves in plasma

(a) $\left(\varepsilon(\omega)\cdot\omega^2 - c^2\vec{k}^2\right)\vec{E}(\omega,\vec{k}) = 0$, in Fourier components

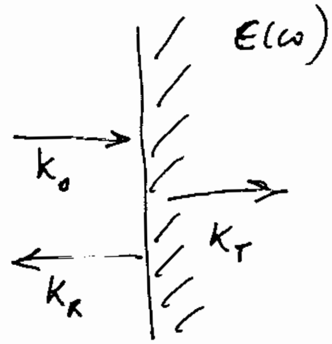
$\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$,

$$\varepsilon(\omega)\,\omega^2 - c^2\vec{k}^2 = 0 \Rightarrow \boxed{\omega^2 = \omega_p^2 + c^2\vec{k}^2}$$

$\omega \geq \omega_p$

(b) $\boxed{v_g(k) = \dfrac{\partial\omega(k)}{\partial\vec{k}} = \dfrac{c^2\vec{k}}{\sqrt{\omega_p^2 + c^2\vec{k}^2}} \text{ - group velocity}}$

(c)



$k_0 = -k_R = \dfrac{\omega}{c}$, $\quad k_T = \dfrac{\omega}{c}\sqrt{\varepsilon}$

$\vec{E}_{\parallel}, \vec{H}_{\parallel}$ are continuous;

$$\left\{\begin{array}{l} E_0 + E_R = E_T \\ k_0(E_0 - E_R) = k_T E_T \end{array}\right.$$

$$\left\{\begin{array}{l} E_0 + E_R = E_T \\ E_0 - E_R = \sqrt{\varepsilon}\,E_T \end{array}\right. \Rightarrow \boxed{\begin{array}{l} E_T = \dfrac{2}{1+\sqrt{\varepsilon}}E_0 \\ E_R = \dfrac{1-\sqrt{\varepsilon}}{1+\sqrt{\varepsilon}}E_0 \end{array}}$$

$\omega < \omega_p$ $\quad R = \left|\dfrac{E_R}{E_0}\right|^2 = \left|\dfrac{1 - i\sqrt{-\varepsilon}}{1 + i\sqrt{-\varepsilon}}\right|^2 = 1 \rightarrow$ total reflection

$$E_w(t) = E_0(\cos\omega t'\hat{x} \pm \sin\omega t'\hat{y}),\quad B_w = n\times E_w$$
$$t' = t - z/c$$

Problem 3

a) $$m\ddot{\vec{r}} = eE_w(t) + \frac{e}{c}\dot{r}\times(\vec{B}+\vec{B}_w)$$

For non relativistic electron, drop Lorentz force due to B_w, since

$$\left|\frac{1}{c}(\dot{r}\times B_w)\right| \approx \frac{v}{c}|B_w| \ll |E_w|$$

Circular motion in x-y plane

$$r(t) = r_0\left(\cos(\omega t+\varphi)\hat{x} \pm \sin(\omega t+\varphi)\hat{y}\right)$$

$$-\omega^2 m r_0\left(\cos(\omega t+\varphi), \pm\sin(\omega t+\varphi)\right) = eE_0(\cos\omega t, \pm\sin\omega t)$$
$$+e\frac{\omega r_0}{c}B\left(\pm\cos(\omega t+\varphi), \sin(\omega t+\varphi)\right)$$

Can satisfy eqs by $\underline{\varphi = 0}$

$$-\omega^2 m r_0 = eE_0 \pm \frac{eB}{c}\omega r_0 \rightarrow \boxed{r_0 = \frac{e}{m}\frac{E_0}{\mp\omega\omega_c - \omega^2} \text{ (amplitude)}}$$

b) Total radiated power:

$$P = \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{eE_0\omega^2/m}{\omega^2\pm\omega\omega_c}\right)^2 = \frac{2}{3}\frac{e^4\omega^4E_0^2}{c^3(\omega^2\pm\omega\omega_c)^2}$$

Energy flow

$$\vec{S} = \frac{c}{4\pi}E_w\times B_w = \frac{c}{4\pi}E_0^2\vec{n}$$

cross-section of radiation

$$\underline{\sigma = \frac{8\pi}{3}\frac{e^4}{m^2c^4}\frac{\omega^2}{(\omega\pm\omega_c)^2}}$$