

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J

April 12, 2006

Problem Set 9

Due: Friday, April 21 (NOT TO BE TURNED IN)

Reading: Zeilik & Gregory: Finish reading Chapter 17 and start Chapter 18.

Reminder: Quiz #2 will be given out on Wednesday, April 19, as a 2-hour take-home exam. You may use a calculator and two pages of prepared notes. The quiz is to be turned in at the lecture on Friday, April 21. If you are not able to work the exam during this time period, please see the instructor for an alternate schedule. A practice exam is available in the exams section.

Problem 1

“Dimensional Analysis of Equations of Stellar Structure”

Zeilik & Gregory, Chapter 16, Problem 5, page 330 (parts **a**, **b**, and **c** only).

Parts (a) and (b) were sketched out in lecture. For part (b) refer to Z&G equation 16-12a and note that $(\kappa\rho)^{-1}$ is defined to be the mean free path (λ_{mfp}) that we used in lecture. Adopt the Z&G suggestion that κ is taken to be a constant. To complete part (b) you should recall the relation $T_c \propto M/R$ that we derived in lecture [using the ideal gas law and the answer to part (a)]. To derive part (c), use the result of part (b) and the fact that $L \propto R^2 T_e^4$, where T_e is the effective surface temperature of the star. To eliminate R , use the empirical fact that $M \propto R$ for stars on the main sequence. Z&G are confusing the central temperature with the surface temperature, and therefore whatever scheme they are looking for in part (c) is likely wrong.

Problem 2

“Spectroscopic Parallax for a Globular Cluster”

The color/apparent magnitude diagram given on the last page is for the globular cluster M3.

- Use the tabulated relation between $(B - V)$ color and absolute visual magnitude (see attached Table) and the H-R diagram for M3 (see attached figure) to calculate an approximate distance to M3.
- From the color of stars at the “turnoff” point, deduce the largest mass of stars that are still on the main sequence. Use the Tables below the H-R diagram for M3 to find the relation between the color and the stellar mass of stars that are on the main sequence (luminosity class V). [To obtain a more accurate answer you must correct for the fact that main-sequence stars in globular clusters are *bluer* than their counterparts in the galactic plane because of

their low metallicity (small Z). For low-mass stars this effect amounts to a *smaller* $B - V$ color (for the cluster stars) by ~ 0.4 magnitudes.]

Problem 3

“Ascending the Giant Branch”

a. Suppose the time-averaged luminosity of a $1 M_{\odot}$ star after it leaves the main sequence is $\sim 20L_{\odot}$. Suppose further that in ascending the giant branch the helium core increases by about $0.3M_{\odot}$. How long does it take such a star to ascend to the tip of the red-giant branch? Express your answer in billions of years. Recall that the efficiency of hydrogen “burning” is $\Delta E/(mc^2) = 0.007$. [for reference, $L_{\odot} \simeq 4 \times 10^{33}$ ergs s^{-1}]

b. Suppose we are considering a large region of the galaxy in which star formation is, on average, continuously occurring over a very long period of time. In such a region (unlike in a single cluster), the Hertzsprung–Russell diagram would *not* evolve with time. What ratio of red giant stars to main sequence stars is to be expected for $1 M_{\odot}$ stars? (Hint: a $1 M_{\odot}$ star spends approximately 7×10^9 years on the main sequence).

Problem 4

“Mean Density of Collapsed Stars”

Zeilik & Gregory, Chapter 17, Problem 5, page 349.

Problem 5 **Optional**

“Cooling White Dwarf”

Zeilik & Gregory, Chapter 17, Problem 6, page 349–350.

[Set the luminosity L equal to $4\pi\sigma R^2 T^4$. For the number density use ρ/μ , where μ is the average mass of a particle: $\sim 1.6 \times 10^{-24}$ g. You will end up with a simple differential equation which can be integrated almost by inspection. The limits on the temperature integral are 10^9 K to T , while the limits on the time integral go from $t = 0$ to $t = 10^{10}$ years.]

Problem 6

“Maximum Rotation Rate for a Pulsar”

How fast can a neutron-star rotate before it breaks up?

a. Find an expression for the minimum rotation period, P_{\min} , of a neutron star as a function of its mass, M , and radius, R .

b. Evaluate P_{\min} for a neutron star with $M = 1.4M_{\odot}$ and $R = 10$ km.

[Hint: Find the rotation rate at which a small parcel of matter near the surface of the neutron star and near its equator would have a centrifugal force equal to the gravitational force on it.]

Problem 7

“Pulsar Spin-Down”

The Crab Nebula has a total luminosity of $\sim 5 \times 10^{38}$ ergs/sec. This supernova remnant is powered by a centrally located pulsar which is now rotating with a period of 33 milliseconds. The pulsar is observed to be slowing down (period is increasing), and the corresponding decrease in its supply of stored rotational kinetic energy is used to inject power into the Crab Nebula. Assume a mass and radius for the neutron-star pulsar of $1.4M_{\odot}$ and 10 km, respectively.

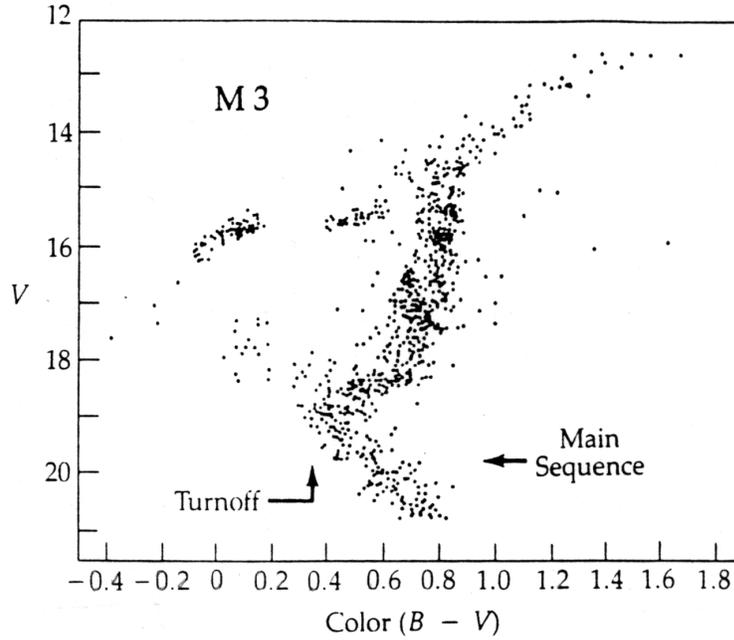
- a. Write down an expression for the rotational kinetic energy ($I\omega^2/2$) in terms of the mass of the neutron star, M , the radius, R , and the rotation period, P . [Approximate the moment of inertia, I , as that for a uniform density sphere, i.e., $2MR^2/5$.]
- b. Compute how much energy is stored in the Crab pulsar in the form of rotational kinetic energy.
- c. Differentiate your answer to part (a) to find a general expression for the rate at which rotational kinetic energy is lost as the period of a pulsar increases (i.e., as the rotation rate of the pulsar decreases). Express your answer in terms of R , M , P , and dP/dt .
- d. Evaluate this expression for the Crab pulsar, given the fact that the observed spin-down rate (dP/dt) amounts to 36 nanoseconds per day. Compare this to the power output of the entire Crab Nebula.
- e. Use the spin-down rate given in part (d) to make a rough estimate of the age of the Crab pulsar and hence that of the nebula. [Hint: $\text{age} \simeq P \div (dP/dt)$] Compute the actual age from the fact that the supernova that led to the formation of the pulsar was observed by the Chinese in the year 1054 A.D.

Problem 8 **Optional**

“Blackbody Radiation from a Neutron Star”

Zeilik & Gregory, Chapter 17, Problem 13, page 350 (part **a** only).

In certain circumstances, where a neutron star is accreting large quantities of matter from a companion star, its surface temperature may be heated to 2×10^7 K. Compute the blackbody luminosity in such a case.



Stellar colours, temperatures, and bolometric corrections

Notation is from § 93. T_v refers to visible region. M_v is included for easy reference to § 95.

Sp	M_v	$B-V$ [1, 4, 5, 6, 7, 8]	$U-B$	T_v	T_v	B.C. [1, 2, 3, 4]	M_{bol}
				[1, 2, 9, 10, 12]	[1, 2, 9, 10, 12]		
				°K	°K		
Main sequence, v							
O5	- 6	-0.45	-1.2	35 000	70 000	4.6	-10.6
B0	- 3.7	-0.31	-1.07	21 000	38 000	3.0	- 6.7
B5	- 0.9	-0.17	-0.56	13 500	23 000	1.6	- 2.5
A0	+ 0.7	0.00	0.00	9 700	15 400	0.68	0.0
A5	+ 2.0	+0.16	+0.09	8 100	11 100	0.30	+ 1.7
F0	+ 2.8	+0.30	+0.02	7 200	9 000	0.10	+ 2.7
F5	+ 3.8	+0.45	-0.01	6 500	7 600	0.00	+ 3.8
G0	+ 4.6	+0.57	+0.04	6 000	6 700	0.03	+ 4.6
G5	+ 5.2	+0.70	+0.20	5 400	6 000	0.10	+ 5.1
K0	+ 6.0	+0.84	+0.46	4 700	5 400	0.20	+ 5.8
K5	+ 7.4	+1.11	+1.06	4 000	4 500	0.58	+ 6.8
M0	+ 8.9	+1.39	+1.24	3 300	3 800	1.20	+ 7.6
M5	+12.0	+1.61	+1.19	2 600	3 000	2.1	+ 9.8

Mass, radius, luminosity, and mean density with spectral class

s-g = supergiant, g = giant, d = dwarf

A single column between g and d represents the main sequence

Sp	$\log(M/M_\odot)$			$\log(R/R_\odot)$			$\log(L/L_\odot)$			$\log \bar{\rho}$		
	s-g	g	d	s-g	g	d	s-g	g	d	s-g	g	d
	[1, 2, 3, 4, 5, 6]			[1, 2, 7]						in $g\text{ cm}^{-3}$		
O5	+2.2		+1.60			+1.25			+5.5			-2.0
B0	1.7		1.23	+1.3	+1.2	0.88	+5.5		4.1	-2.1		-1.3
B5	1.4		0.85	1.5	1.0	0.60	4.9		2.8	-2.9		-0.8
A0	1.2		0.55	1.6	0.8	0.42	4.4		1.9	-3.5		-0.6
A5	1.1		0.34	1.7		0.25	4.1		1.3	-3.8		-0.3
F0	1.1		0.25	1.8		0.13	3.9		0.8	-4.2		0.0
F5	1.0		0.15	1.9	0.6	0.08	3.8		0.4	-4.5		+0.1
G0	1.0	+0.4	+0.03	2.0	0.8	+0.02	3.8	+1.5	+0.1	-4.9	-1.8	+0.1
G5	1.1	0.5	-0.03	2.1	1.0	-0.03	3.8	1.7	-0.1	-5.2	-2.4	+0.2
K0	1.1	0.6	-0.09	2.3	1.2	-0.07	4.0	2.0	-0.4	-5.7	-2.9	+0.3
K5	1.2	0.7	-0.16	2.6	1.4	-0.13	4.3	2.3	-0.8	-6.4	-3.4	+0.4
M0	1.2	0.8	-0.32	2.7		-0.20	4.5	2.6	-1.2	-6.7	-4	+0.5
M2	1.3		-0.40	2.9		-0.3	4.7	2.8	-1.5	-7.2		+0.7
M5			-0.65			-0.5		3.0	-2.1			+1.1
M8			-1.1			-0.9			-3.3			+1.7