

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J

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Problem Set 11

Due: Friday, May 5 (in lecture)

Reading: Zeilik & Gregory, Chapters 19, 20, and start 21

Problem 1

“X-Ray Burst Source”

A neutron star accretes 10^{21} grams of hydrogen-rich matter from a binary companion star during the 5-hour interval since its last burst. The accreted matter forms a thin spherical shell on the surface of the neutron star. At some instant there is runaway thermonuclear burning of the accreted matter; all the fusion reactions are completed in $\ll 1$ second. Assume that the fraction of the rest mass of the accreted matter that is converted to energy is equal to 0.003. As a result of the energy released, the surface of the neutron star becomes very hot and it radiates away this excess energy in the form of black body radiation. Take the radius of the neutron star to be 10 km.

- How much energy is released in the burst?
- If the burst lasts for about 10 sec, what is the average luminosity of the burst? Compare this to the luminosity of the Sun.
- Assume that during the burst the surface of the neutron star reaches a temperature T which remains nearly constant over an interval of ~ 10 sec. Find T .
- Use the Wien displacement law (page 172 in Z&G) to demonstrate why these events are called “X-ray bursts”.

Problem 2

“Accretion Powered X-Ray Source”

While studying a typical X-ray source in the Andromeda galaxy, the X-ray telescope on the *Chandra* observatory detected 300 X-rays, each of energy ~ 3 keV ($\sim 5 \times 10^{-9}$ ergs), during a 10,000 sec observation. Assume that these X-rays result from accretion of matter onto a neutron star that is in a binary system within the Andromeda galaxy (distance ~ 700 kpc). Further assume that almost all of the accretion energy is emitted in the X-ray region of the spectrum (near ~ 3 keV). You also need to know that the effective collecting area of the X-ray telescope is ~ 1000 cm².

- Make an estimate of the X-ray luminosity of this source.
- Compute an approximate mass accretion rate (in $M_{\odot} \text{ yr}^{-1}$) that is required to supply the

power for the X-ray production. Assume a neutron star mass and radius of $1.4 M_{\odot}$ and 10 km, respectively.

Problem 3

“Interstellar Extinction”

Zeilik & Gregory, Chapter 15, Problem 1, page 307.

[Hint: The intrinsic color of such a star is $B - V = 0$, while the absolute visual magnitude, $M_V = +0.7$]

Problem 4 **Optional**

“21-cm Hydrogen Radiation”

Zeilik & Gregory, Chapter 15, Problem 8, page 308.

[Hint: The spontaneous decay timescale for a hydrogen atom to emit 21-cm radiation is about 10^7 years.]

Problem 5

“Distance to a Dark Cloud”

Zeilik & Gregory, Chapter 15, Problem 15, page 308.

Problem 6

“Short Problems”

Zeilik & Gregory, Chapter 15, Problems 5 and 10, page 308.

[Notes: In Problem 5, “absorption lines” refers to the visible region of the spectrum; for Problem 10, the Balmer formula can be found on page 161 of Zeilik & Gregory]

Problem 7

“Short Problems”

Zeilik & Gregory, Chapter 15, Problems 4 and 12, page 308.

Problem 8 **Optional**

“Planetary Nebula”

Near the end of its life, a star may blow off its outer envelope leaving only a hot remnant “white dwarf” star. The ultraviolet light from the hot central star can then fluoresce the matter that has been ejected. The result is called a “planetary nebula”, an example of which is shown in Figure 15-11 on page 294 of Z&G.

a. Assume that these objects lost their envelopes approximately 30,000 years ago and that the ejected matter has been expanding at a rate of 25 km s^{-1} ever since. Compute the present radius, R_s , of the expanding shell, which we take to be spherically symmetric.

- b. If the mass of the ejected envelope is $1/2 M_{\odot}$ ($\simeq 10^{33}$ g) find the average number density of atoms within the shell at the present time. Assume that the shell is composed of pure hydrogen ($m_H = 1.66 \times 10^{-24}$ g) and that there is a uniform density, n , everywhere within the radius R_s .
- c. Compute the bolometric luminosity of the central star if its radius and temperature are 10^9 cm and 50,000 degrees K, respectively.
- d. If one half of the luminosity of the central star is emitted at wavelengths shorter than the Lyman limit (916 Å), estimate the number of photons per second that are emitted with $\lambda < 916$ Å (photon energy > 13.6 eV = 2.2×10^{-11} ergs).
- e. Use the result of part (d) and the formalism that was developed in lecture to understand Stromgren spheres to estimate the radius of the nebula that can be fluoresced (“lit up”) by the central star. How does the calculated radius of the optically bright nebula compare to the radius R_s calculated above. [Useful relation: the rate (per unit volume) at which electrons and protons recombine to form neutral hydrogen is given by $n_H^2 \alpha$, where $\alpha = 3 \times 10^{-13}$ cm⁻³ s⁻¹.]