Last time considered effects of Cal-Baron field:

Action S:

$$S = \frac{1}{\sqrt{2\pi\alpha'}} \int d\tau d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 (X')^2} - \frac{1}{2} \int d\tau d\sigma \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \sigma} B_{\mu\nu}(x) - \frac{1}{6k^2} \int d^D x H_{\mu\nu\rho} H^{\mu\nu\rho}$$

First term: String action
Second term: Interaction
Third term: Coupling to current

Interaction term rewritten as: $-\int d^D x B_{\mu\nu}(x) J^{\mu\nu}(x)$ where current $J^{\mu\nu}(x) = \int d\tau d\sigma \frac{\partial x}{\partial \tau} \frac{\partial x}{\partial \sigma} \cdot \delta(x - X(\tau, \sigma))$

$$\boxed{\frac{1}{k^2} \frac{\partial H^{\mu\nu\rho}}{\partial x^\rho} = J^{\mu\nu}}$$

Antisymmetric in ρ and ν.

$$J^{\mu\nu} = -J^{\nu\mu}$$

$$\frac{\partial}{\partial x^\mu} \Rightarrow \boxed{0 = \frac{\partial J^\mu}{\partial x^\mu}}$$

Conservation index μ so we seem to have a collection of conserved currents.

$J^{\mu\nu}$ μ: conservation index, ν: labels for various currents

J^{0k}: string charge densities (to find charge, integrate over space). B_{0k} couples to J_{0k}.

Conservation equation for $\nu = 0$:

$$\frac{\partial J^{\mu 0}}{\partial x^\mu} = 0 \Rightarrow \frac{\partial J^{0\mu}}{\partial x^\mu} = 0 \Rightarrow \frac{\partial J^{0k}}{\partial x^k} = 0$$

$\vec{J}^o = (J^{01}, J^{02}, \ldots, J^{0d})$. ($\vec{J}^0$ is a vector), so:

$$\frac{\partial J^{0k}}{\partial x^k} = 0 \Rightarrow \frac{\partial (\vec{J}^0)^k}{\partial x^k} = 0 \Rightarrow \boxed{\nabla \cdot \vec{J}^0 = 0}$$

String charge lives only on the string. Magnetic fields help us, but not a perfect analogy. Perfect analogy: string charge is like stationary electric currents. Remember current conservation from $E\&M$:

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Stationary currents have $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{J} = 0$ e.g. a current on a closed loop, or an infinite wire. A current that, e.g. ends at a capacitor to charge it is not stationary. Open strings are problematic: charge flowing in string accumulates at ends.

$$J^{0k}(t, \vec{x}) = \frac{1}{2} \int d\tau d\sigma \left(\frac{\partial x^0}{\partial \tau} \frac{\partial x^k}{\partial \sigma} - \frac{\partial x^k}{\partial \tau} \frac{\partial x^0}{\partial \sigma} \right) \cdot \delta(t - x^0(\tau, \sigma)) \cdot \delta(\vec{x} - \vec{x}(\tau, \sigma))$$

Static gauge, $x^0 = \tau$

$$\begin{aligned} J^{0k} &= \frac{1}{2} \int d\sigma \delta(\vec{x} - \vec{x}(t, \sigma)) \frac{\partial x^k}{\partial \sigma} \\ J^0 &= \frac{1}{2} \int d\sigma \delta(\vec{x} - \vec{x}(t, \sigma)) \frac{\partial x}{\partial \sigma} \end{aligned}$$

If we have a closed string, we'll have a string charge density vector everywhere along the string. Now we know the direction of this vector: along the direction of increasing σ. String charge behaves like stationary electric current. J^{ik} for a static string is 0. It has to do with the velocity of the string.

Consider a static string in $3 + 1$ dimensions (could be an infinitely long string - interesting and simple case). Must solve:

$$\frac{1}{\kappa^2} \frac{\partial H^{\mu\nu\rho}}{\partial x^\rho} = J^\mu$$

Assume all H's are time independent with $H^{ijk} = 0$.

Static string, so really two equations (μ, ν are i, j)

$$\frac{1}{\kappa^2}\frac{\partial H^{ij\rho}}{\partial x^\rho} = J^{ij} = 0$$
$$\frac{\partial X^{ij0}}{\partial x^0} + \frac{\partial H^{ijk}}{\partial x^k} = 0$$

Expanding over ρ index, which can take values 0 and spatial.

$$\frac{1}{\kappa^2}\frac{\partial H^{0\nu\rho}}{\partial x^0} = J^{0\nu}$$

Totally antisymmeteric, so can't have other 0 indices, so all other indices spatial.

$$\frac{1}{\kappa^2}\frac{\partial H^{0kl}}{\partial x^l} = J^{0k}$$

Let's recast this equation as something more familiar. Let:

$$H^{0kl} = \kappa^2 \epsilon^{k0m} B^m$$

Recall $\epsilon^{123} = 1$. Any reversal of index order changes its sign.

Plug in:

$$\frac{1}{\kappa^2}\frac{\partial}{\partial x^l}(\kappa^2 \epsilon^{k0m} B^m) = J^{0k}$$

$$\epsilon^{klm}\partial_l B^m = J^{0k}$$

This is the familiar $(\nabla \times B)^k = (\vec{J}^0)^k$

$$\nabla \times B = \vec{J}^0$$

So finding Kalb-Raman field of a magnetic field is mathematically equivalent to finding the electric current.

We have $H^{\mu\nu\rho}$ whose E&M analogue is $\vec{B}$ (the magnetic field). $B^{\mu\nu}$ whose E&M analogue is $\vec{A}$ (vector potential). J^{0k} whose E&M analogue is $\vec{J}$ (current).

Action for coupling to Kalb-Raman field:

$$\begin{aligned} S_B &= -\int d\tau d\sigma \frac{\partial x^\mu}{\partial \tau}\frac{\partial x^\nu}{\partial \sigma} B_{\mu\nu}(x) \\ &= -\frac{1}{2}\int d\tau d\sigma \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}(x) \end{aligned}$$

where $\alpha, \beta \in 1, 2$ and $\epsilon^{12} = 1$, $\epsilon^{21} = -1$, and $\partial_1 = \partial_\tau$, $\partial_2 = \partial_\sigma$. α and β are coordinate indices on the worldsheet.

$q\int A_\mu dx^\mu$: coupling of E&M to a point charge. Gauge invariant? Yes! Things made with A have a hard time being gauge invariant. Reason above is gauge invariant.

$$\begin{aligned} q\int A_\mu dx^\mu &= q\int A_\mu(x(\tau))\frac{\partial x^\mu}{\partial \tau} d\tau (\text{Let } \delta A_\mu = \delta_\mu \epsilon) \\ &= q\int \frac{\partial \epsilon(x(\tau))}{\partial x^\mu}\frac{\partial x^\mu}{\partial \tau} d\tau \\ &= q\int \frac{\partial \epsilon}{\partial \tau}(x(\tau)) d\tau \\ &= q(\epsilon(\tau = \infty) - \epsilon(\tau = -\infty)) \end{aligned}$$

Gauge invariant if ϵ vanishes in ∞ past or ∞ future. Good enough!

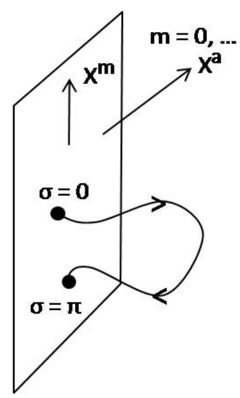
$$\begin{aligned} \delta B_{\mu\nu} &= \partial_\nu \Lambda_\nu - \partial_\nu \Lambda_\mu \\ \delta_{B\mu\nu}(x) &= \frac{\partial \Lambda_\nu}{\partial x^\mu} - \frac{\partial \Lambda_\mu}{\partial x^\nu} \end{aligned}$$

$\Lambda = \Lambda(x)$

$$\begin{aligned}
\delta S_B &= -\int d\tau d\sigma \epsilon^{\alpha\beta}\left(\frac{\partial \Lambda_\nu}{\partial x^\mu}\right)\partial_\alpha x^\mu \partial_\beta x^\nu \\
&= -\int d\tau d\sigma \epsilon^{\alpha\beta}\epsilon^{\alpha\beta}\partial_\alpha \Lambda_\nu \partial_\beta x^\nu \\
&= -\int d\tau d\sigma \partial_\alpha(\epsilon^{\alpha\beta}\Lambda_\nu \partial_\beta x^\nu) \\
&= \int d\tau d\sigma \frac{\partial}{\partial\sigma}(\Lambda_\nu \partial_\tau x^\nu)
\end{aligned}$$

$$\delta S_B = \int d\tau[\Lambda_\nu \partial_\tau x^\nu_{\sigma=\pi} - \Lambda_\nu \partial_\tau x^\nu_{\sigma=0}]$$

Consider now a string ending on a D-brane. What kind of violation of the gauge invar. will we get?



$$\Lambda_\nu \partial_\tau x^\nu = \Lambda_m \partial_\tau x^m + \underbrace{\Lambda_a \partial_\tau x^a}_{0 \text{ since } x^a \text{ at string ends is constant}}$$

$$\delta S_B = \int d\tau[\Lambda_m \partial_\tau x^m_{\sigma=\pi} - \Lambda_m \partial_\tau x^m_{\sigma=0}]$$

String cannot end on the D-brane.

We've accumulated so much evidence that this makes sense, but then we get stuck. Here's where we need inspiration:

Possible hints: String conservation, charge conservation. Maybe string ends not so innocent - maybe they're electrically charged (this would be good - then string theory would include electric charge, a very necessary element of a physical theory).

Approach: Let's believe string endpoints are charged. Say $\sigma = \pi$ has a positive charge, $\sigma = 0$ has a negative charge.

$$S = S_B + \int d\tau A_m(x) \frac{\partial x^m}{\partial \tau}_{\sigma=\pi} - \int d\tau A_m(x) \frac{\partial x^m}{\partial \tau}_{\sigma=0}$$

$$\delta B_{\mu\nu} = \partial_\mu \partial_\nu - \partial_\nu \partial_\mu$$
$$\delta B_{mm} = \partial_m \Lambda_n - \partial_n \Lambda_m$$
$$\delta A_m = -\Lambda_m$$

Kalb-Maron parameter is changing the magnetic field. Outrageous, but necessary. Then get gauge invariant.

Such a strong gauge transformation, might wonder if our cure is worse than the disease. Not so:

$$S = S_B + S_{EM} \text{ is gauge invariant}$$

But what have we done to Maxwell? Not so severe.

F_{mn} used to be gauge invar. Now?

$$\delta F_{mn} = \delta(\partial_m A_n - \partial_n A_m) = -\partial_m \Lambda_n + \partial_n \Lambda_m$$

Not gauge invariant, but

$$\delta F_{mn} = -\delta B_{mn}$$

So $F_{mn} + B_{mn}$ is gauge invariant. So in string theory with Kalb Raman fields, can't just use F, must use:

$$\boxed{\Im_{mn} = F_{mn} + B_{mn}} \qquad \text{(gauge invar.)}$$

This looks good. B_{mn} (gravitational, field of closed string) very small usually. So practically $\Im_{mn} \approx F_{mn}$ most of the time.

Before:

$$S_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Now must write:

$$S_{EM} = -\frac{1}{4} \Im_{mn} \Im^{mn} = -\frac{1}{4} F_{mn} F^{mn} - \frac{1}{4} B_{mn} B^{mn} - \frac{1}{2} F_{mn} B^{mn}$$

Recall string charge $B^{0k} J_{0k}$. So F_{0k} couples to B^{0k} - electric fields now have string charge!

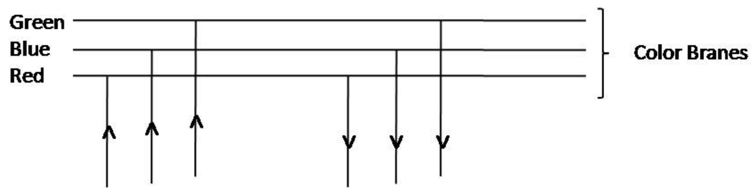
Charge at string endpoints create electric fields in the brane that continue to carry string charge.

Maybe string is made of electric field lines all bunched up together that can fly in/out on the brane.

But strings not *just* made of field lines ... doesn't quite work.

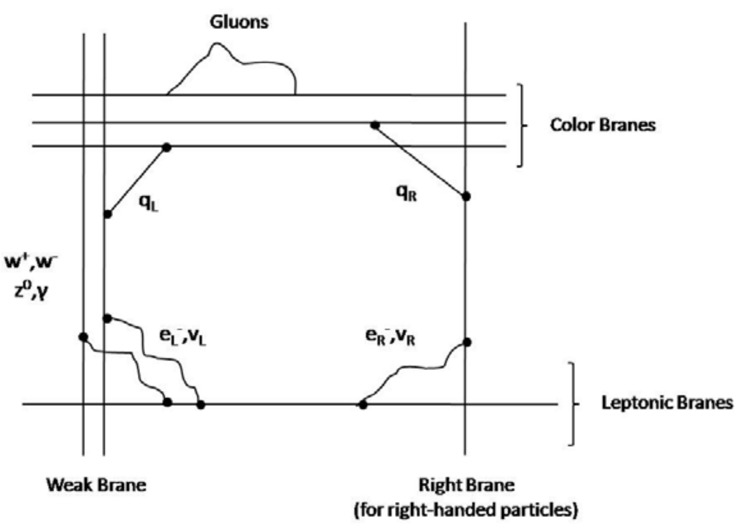
This model implies that particle-antiparticle annihilation are a closed string going off the brane.

Quarks and QCD: Consider 3 D-branes (for 3 colors):

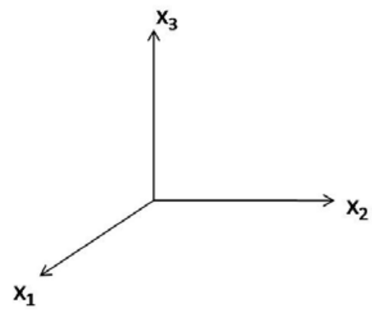


1. Open string ending on the red brane is a red quark.
2. Blue quark.
3. Green quark.
4. Red anti-quark. (going away)

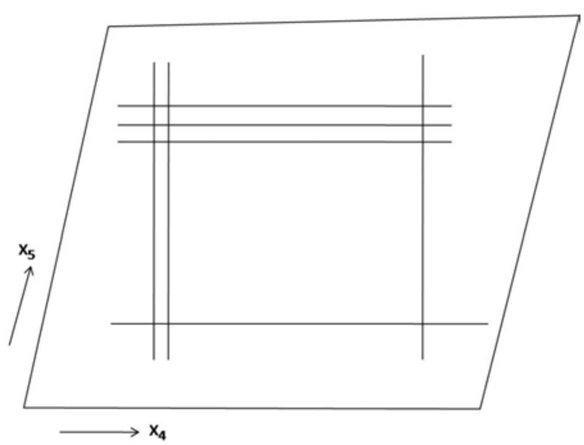
Add in weak brane and leptonic branes.



Call all these branes D4 branes filling world.



But string theory has extra dimensions. But all those branes on a torus.



This model found to support all the particles we know and more we don't. Not a complete story since we need symmetry-breaking. Maybe there's supersymmetry, so need some. Higgs boson from tachyons from intersecting branes. Not a 100% model, but a good first attempt.