### 8.251 - Homework 5

Due Tuesday, March 13.

1. (10 points) Problem 6.2 (restated here for your convenience with added notation).

Examine the Nambu-Goto action (6.39) for a relativistic string with endpoints attached at ( $0, \overrightarrow{0}$ ) and $(a, \overrightarrow{0})$. Consider the non-relativistic approximation where $\left|\vec{v}_{\perp}\right| \ll c$ and the oscillations are small (see (4.3), whose left-hand side should have an absolute value!).
You may denote by $\vec{y}$ the collection of transverse coordinates $X^{2}, \ldots X^{d}$ and write $\vec{y}(t, x)$, where $x$ is the coordinate corresponding to $X^{1}$.
Work in the static gauge. Moreover, parameterize the strings using $X^{1}=x=a \sigma / \sigma_{1}$. This parameterization is allowed for small oscillations. In fact, it is allowed for any motion in which $X^{1}$ is an increasing function along the string.

Show that the action reduces, up to an additive constant, to the action for a non-relativistic string performing small transverse oscillations. What is the tension and the linear mass density of the resulting string? What is the additive constant?
2. (5 points) Consider a D1-brane in four-dimensional spacetime. The brane lies on the $x^{3}=0$ plane and it is rotated an angle $\theta$ with respect to the $x^{1}$ axis in the counterclockwise direction. Describe in full detail the boundary conditions that apply to open strings on this D-brane. Give your answers as a set of conditions on $X^{\mu}\left(\tau, \sigma_{*}\right)$ and $\mathcal{P}_{\mu}^{\sigma}\left(\tau, \sigma_{*}\right)$, where $\mu=0,1,2,3$.
3. (10 points) Problem 6.4.
4. (10 points) Problem 6.6.
5. (15 points) Problem 6.7.
6. (10 points) Problem 7.3.
7. (10 points) Problem 7.4.

I would recommend problem 7.2 for practice.

