PROFESSOR: Hydrogen atom, the first thing to do is to describe the potential V or r. And it would be in the units that we'd like to use minus e squared over r. e is the charge of the electron, and the electron and proton with the same charge. The potential energy is negative. And that sign you should be comfortable with. It's suggesting that you go closer and closer. You're going down and energy is favored. The two particles to go on top of each other.

Immediately we want to generalize it a little so that it's hydrogen like atoms. And we'll put a Z here, assuming that the proton, instead of the program, there's a nucleus with z protons. And there probably would be about z electrons, but we're worried now about just one electron.

And in this case, a charge in the proton is ze multiplied by the charge of the electron gives you this as the potential energy. And this is advantageous. You sometimes have an alpha particle that captures an electron, and then there's two protons. And you don't want to solve this, again, from the beginning. So just put the z, and that's what we will do.

So a few numbers. We've done some of these numbers before, but the Bohr radius and one way of calculating the Bohr radius is to just think of units and think of energy. Energy goes like h squared over ma squared. This has units of energy. You remember p squared over 2m. And p is h over distance.

So if you have a Bohr radius a 0, this quantity has units of energy. But a potential has units of energy and we just put e squared over a0. So this is a consistent equation between two quantities that have units of energy from which you can get the unique length that has units of energy, which is a0. And a0 is h squared over me squared. It's a very simple and nice constant is the Bohr radius.

Intuitively one thing that should remember, e is appearing in the right place. And you could imagine if the strength of electricity was weaker and weaker, like setting e going to 0, the atom would become bigger and bigger. It would just not be able to hold it. So it's reasonable to expect this to happen.

So at this moment, we can calculate what this is, at least, estimate it. And for that, we multiply by a c squared e squared over m-- mc squared. And then we recall that e is squared over hc is about 1 over 137. So we write this as hc over e squared over hc times mc squared.

Now in here what mass is the mass that we should put. I will not be all that careful. It should really be the reduced mass. But in differs by a factor of one part in 1,000 or less even from the mass of the electrons. So I'll put just the mass of the electron.

This it's about 197 mev for Fermi. This is 1 over 137. And for the electron is 0.5 times 10 to the 6 ev 0.5 mev. I won't run the numbers. The answer is about 0.529 angstroms, which is about 53 picometers.

Angstrom is 10 to the minus 10 meters. Picometer 10 to the minus 12 meters. So that's a length scale you've seen several times. There's an energy scale that is famous to. And that's e squared over a0. Because energy comes here, the energy scale is e squared over e0.

So you can substitute what a0 is because you know it already. And you get e to the fourth over m over h squared, which is e to the fourth over h squared c squared times Mc squared. And you see it's kind of nice to see these quanities appearing, because here you have e squared over hc squared times mc squared.

So the typical energy of the hydrogen atom is the fine-structure constant, sometimes called alpha squared times an energy. And what energy's available in the problem? The rest energy of the electron.

So if the bound state energy should be something, it should be a number proportional to the energy that the problem already has. And the problem has one energy, the rest energy of the electron. So it's not surprising. So it's one over 137 squared times 511,000 ev. And that's about 27.2 ev.

And the reason this may sound familiar is because the true ground state energy of the hydrogen atom is this number divided by 2, which is 13.6 ev. So of course, you would not know at this stage, because you're just doing numbers. I may remind you of things did a long time ago in this course. You calculated a couple of other constants.

And you showed that alpha-- again, the fine-structure constant comes a0 was the so-called Compton wavelength of the electron, with a bar. So it's h bar over mc. Remember, that the Broglie wavelength is h over the momentum. The Compton wavelength this h over mc. And the Bard can wave at the h bar over mc. And that's what alpha times a zero is.

And that quantity is about 400 Fermi. It's already much smaller than the Bohr radius. It's smaller by 137 from the Bohr radius. And then if you do alpha squared a0, that actually was the classical electron radius.

So you must divide this by 137, again, and it gives you about 2.8 Fermi. And what you can remember is that the size of a proton is about a Fermi. So that gives you a little bit of intuition. So those are the basic numbers that we begin with, with the hydrogen atom. It gives you a scale of what's going on, the size of an atom, and the energies that we're supposed to get.