PROFESSOR: Today's lecture continues the thing we're doing with scattering states. We send in a scattering state. That is an energy eigenstate that cannot be normalized into a step barrier.

And we looked at what could happen. And we saw all kinds of interesting things happening. There was reflection and transmission when the energy was higher than the barrier. And there was just reflection and a little exponential decay in the forbidden region if the energy was lower than the energy of the barrier.

We also observed when we did the packet analysis that a wave packet sent in would have a delay in coming back out. It doesn't come out immediately. And that's the property of those complex numbers that entered into the reflection coefficient. Those complex numbers were a phase that had an energy dependence. And by the time you're done with analyzing how the wave packet is moving, there was a delay.

So today we're going to see another effect that is sometimes called resonant transmission. And it's a rather famous. Led to the so-called Ramsauer-Townsend effect. So we'll discuss that. And then turn for the last half an hour into the setup where we can analyze more general scattering problems.

So let's begin with this Ramsauer-Townsend effect. And Ramsauer Townsend effect. Before discussing phenomenologically what was involved in this effect, let's do the mathematical and physics analysis of a [INAUDIBLE] problem relevant to this effect.

And for that [INAUDIBLE] problem we have the finite square well. We've normalized this square well. Having width to a. So it extends from minus a to a. That's x equals 0.

And there's a number minus v0. v0 we always define it to be positive. Therefore, v0 is the depth of this potential.

And the question, is what happens if you send in a wave? So you're sending in a particle. And you want to know what will happen to it.

Will it get reflected? Will it get transmitted? What probabilities for reflection, what probabilities for transmission?

So of course, we would have to send a wave packet to represent the physical particle. But we've learned that dealing with energy eigenstates teaches us the most important part of the story. If a particle has some energy, well roughly, it will tend to behave the way an energy eigenstate of that energy does, as far as reflection and as far as transmission is concerned.

So we'll set up a wave. And there's a coefficient A. So there's an Ae to the ikx. That is our wave. And it's moving to the right. Because you remember the time factor, e to the minus iet over h bar. And when you put them together, you see that it's moving to the right.

But presumably, there will be a reflection here. The wave comes in, and some gets reflected and some gets transmitted. And the part that gets transmitted probably will get partially reflected back here and partially transmitted forward.

But the part that is partially reflected will get again reflected here and partially transmitted back. It seems like a never ending process of which, if you think physically what's happening, there's a reflection at the first barrier.

Now you say, wait a moment. Why would there be a reflection? Classically, there would never be a reflection. If the potential goes down, the particle would just be able to continue. We had reflection when we had a barrier.

Well in quantum mechanics, any change in the potential is bound to produce a reflection. So yes, if you have a potential like this, like a jumping board, and you come in here, there's a tiny probability that you will be reflected as you come into this potential drop.

So OK, so this will be reflection. So at the end of the day, there might be many bouncings. If you imagine a particle doing this. Some probability of reflection, some of transmission.

But the end of the day, there will be some wave moving to the left here. So we'll represent it by Be to the minus ikx. So in this region, we have A and a wave with amplitude B going this way.

In the middle region, the same will be true. There will be some wave going here, and some wave that bounces due to this reflection. So there will be a-- I don't know what letters I used. I'd better keep the same. C in this direction and D in this direction.

And now I would have Ce to the ikx. Well, that e to the ikx is not quite right. Because k here, k squared represents the energy. k is the momentum and k squared is energy.

So if you have an energy eigenstate-- oh, my picture is very crowded. So I'll do it anyway. A is here. Maybe I'll put C and D here. And now with A and B here I can write this as the energy.

You have a particle with some energy coming in. And there is this wave here and k squared. It's 2mE over h squared. E is positive scattering states. And indeed, E from that equation is h squared k squared over 2m. What do you know?

But at this point, the total energy, kinetic energy of the particle is bigger. If e is replaced by e plus v0, which is the magnitude of this drop. So here there will be a k2x plus De to the minus ik2x. And k2 refers because it's region two, presumably. People use that name. k2 squared will be 2me plus v0 over h squared.

And finally, to the right of the potential square well there will be just one wave. Because intuitively, we should be able to interpret this as some wave that goes through, but has nothing to make it bounce or reflect.

So we try to get the solution, which will have just some wave going in this direction. And it's called Fe to the ikx. And I can go back to the label k because you have the same energy available as kinetic energy you had to the left of the barrier.

OK, so we've set up the problem. The wave function I would have to write it as three expressions. One for x less than a, one for x in between a and minus a, and one for x greater than a. And those are this one, two, and three formulas.

Any questions about this setup so far?

OK. Well, at this moment you will eventually have some practice on that. The thing that you want to do is relate the various coefficients and define some reflection and transmission coefficients. We saw we had to think in terms of probability current. That's the better way to get an idea of what you should call reflection or transmission coefficient.

So we have to be careful with the case of the step potential when we compared the meaning of the wave that was moving to the right. The amplitude divided by the incoming wave was not quite the transmission coefficient. But in this case, the nice thing is that the wave to the left and the wave to the right are experiencing the same potential. So they can be compared directly.

So I will be able to conjecture, it's reasonable to conjecture that we can define reflection and transmission coefficients as follows. We'll have a reflection coefficient should be B over A squared. B represents a reflected wave, A the incoming wave.

The transmission coefficient you may guess that it's F over A squared. And this all will make sense if we have the reflection plus the transmission is equal to 1.

So we have current conservation, conservation. And the current, which is the net probability flow to the left of the barrier or the depression over here, is J on the left is proportional to A squared minus B squared.

You remember, we computed it last time. If you compute the probability current to this Ae to the ikx plus Be to the minus ikx, you get two contributions. Essentially A squared minus B squared. There's a factor of h bar k over m in front.

At any rate, this should be equal to the current that is flowing out in this direction. You see, whatever current is coming in to the left must be the current going out to the right. So this is f squared.

So current conservation really tells you that A squared minus B squared is equal to F squared. And if you pass the B to the other side, you get A squared equal B squared plus F squared. And dividing by A squared you get 1 is equal to B over A squared plus F over A squared. And that's the reflection plus the transmission

So the way we've defined things makes sense. Reflection and transmission are properly defined. And this is because current conservation works well.

So reflection coefficient is essentially the flux in the reflected, the probability current or in the flux in the reflected wave compared to the flux in the incoming wave. The transmission is the probability current or flux of probability in the transmitted wave compared with the one incoming wave.

So you've done all of this set up. Now you can't avoid, however, doing a little bit of calculation, which is boundary conditions. You have one, two, three, four, five variables. And somehow, you want to calculate these ratios.

So you have to say that the wave function and the derivative is continuous at this point. And the wave function and the derivative is continuous and this point. That would give you four conditions.

And that's reasonable. We have five variables. But you know, the overall normalization could never be determined.

So things can be determined in terms of A. So you can expect that with four equations you can solve for B, C, D, and F in terms of A. But the overall scale of this total wave function is undetermined. Its boundary conditions will give you constraints, but it will never determine the magnitude, the overall magnitude of an energy eigenstate.