PROFESSOR: In order to learn more about this subject, we must do the wave packets. So this is the place where you really connect this need solution of Schrodinger's equation, the energy eigenstates, to a physical problem. So we'll do our wave packets.

So we've been dealing with packets for a while, so I think it's not going to be that difficult. We've also been talking about stationary phase and you've practiced that, so you have the math ready. We should not have a great difficulty.

So let's new wave packets with-- I'm going to use A equals 1 in the solution. Now, I've erased every solution, and we'll work with E greater than v naught to begin with. The reason I want to work with E greater than v naught is because there is a transmitted wave. So that's kind of nice.

So what am I going to do? I'm going to write it this following way. So here is a solution with A equals to 1. e to the i kx plus k minus k bar over k plus k bar e to the minus i kx. And this was for x less than 0.

And indeed, when there was an A, there was a B, if A is equal to 1, remember we solve for the ratio of B to A and it was this number so I put it there. I'm just writing it a little differently, but this is a solution. And for x greater than 0, the solution is C, which was 2k over k plus k bar e to the i k bar x.

Now we have to superimpose things. But I will do it very slowly. First, is this a solution of the full Schrodinger equation? No. Is this a solution a time-independent Schrodinger equation? What do I need to make it the solution of the full Schrodinger equation? I need e to the minus i Et over h. And I need it here as well.

And this is a psi now of x and t. There's two options for x, less than 0 and x less than 0, and those are solutions. So far so good.

Now I'm going to multiply each solution. So this is a solution of the full Schrodinger equation, not just the time-independent one, all of it. It has two expressions because there's a little discontinuity in the middle, but as a whole, it is a solution. That is the solution.

Some mathematicians would put a theta function here, theta of minus x and add this with a theta of x so that this one exists for less than x, x less than 0, and this would exist for less greater than 0. I would not do that. I will write cases, but the philosophy is the same.

So let's multiply by a number, f of k. Still a solution. k is fixed, so this is just the number. Now superposition. That will still be a solution if I do the same superposition in the two formulas, integral dk and integral dk. That's still a solution of a Schrodinger equation.

Now I want to ask you what limits I should use for that integral. And if anybody has an opinion on that, it might naively be minus infinity to infinity, and that might be good, but maybe it's not so good. It's not so good. Why is not so good, minus infinity to infinity?

Would I have to do from minus infinity on my force. Does anybody force me? No. You're superimposing solutions. For different values of k, you're superimposing. You had a solution, and another solution for another, another solution. What goes wrong if I go from minus infinity to infinity? Yes.

AUDIENCE: [INAUDIBLE] the wave packet's going to be in one direction, so it [? shouldn't ?] be [? applied. ?]

PROFESSOR: That's right. You see here, this wave packet is going to be going in the positive x direction, positive direction, as long as k is positive. It's just that the direction is determined by the relative sign within those quantities. E is positive in this case. k is positive. This moves to the right.

If I start putting things where k is negative, I'm going to start producing things and move to the left and to the right in a terrible confusion. So yes, it should go 0 to infinity, 0 to infinity.

And f of k, what is it? Well, in our usual picture, k f of k is some function that is peaked around some k naught. And this whole thing is psi of x and t, the full solution of a wave packet.

So now you see how the A, B, and C coefficients enter into the construction of a wave packet. I look back at the textbook in which I learned quantum mechanics, and it's a book by Schiff. It's a very good book. It's an old book. I think was probably written in the '60s. And it goes through some discussion of wave packets and then presents a jewel, says with a supercomputer, we've been able to evaluate numerically these things, something you can do now with three seconds in your laptop, and it was the only way to do this.

So you produce an f of k. You fix their energy and send in a wave packet and see what happens. You can do numerical experiments with wave packets and see how the packet gets distorted at the obstacle and how it eventually bounces back or reflects, so it's very nice. So there is our solution.

Now we're going to say a few things about it. I want to split it a little bit. So lets go here. So how do we split it? I say the solution is this whole thing, so let's call the incident wave that is going to be defined for x less than 0 and t, this is x less than 0. And the incident wave packet is dk 0 to infinity f of k e to i kx e to the minus i et over h bar.

And this is just defined for x less than 0, and that's so important that write it here. For x less than 0, you have an incident wave packet. And then you also have a reflected wave packet, x less than 0 t is the second part dk f of k k minus k bar over k plus k bar e to the minus i kx e to the minus i et over h bar.

It's also for x less than 0, and we have a psi transmitted for x greater than 0 and t, and that would be 0 to infinity dk f of k 2k over k plus k bar e to the i k bar x e to the minus i et over h bar. Lots of writing, but that's important.

And notice given our definitions, the total psi of x and t is equal to psi incident plus psi reflected for x less than 0, and the total wave function of x and t is equal to psi transmitted for x greater than 0. Lots of equations. I'll give you a second to copy them if you are copying them.

So now comes if we really want to understand this, we have to push it a little further. And perhaps in exercises we will do some numerics to play with this thing as well. So I want to do stationary phase approximation here. Otherwise, we don't see what these packets, how they're moving.

So you have some practice already with this. You're supposed to have a phase whose derivative is 0, and it's very, very slowly at that place where there could be a contribution. Now every integral has the f of k. So that still dominates everything, of course.

You see, if f of k is very narrow, you pretty much could evaluate these functions at the value of k naught and get a rather accurate interpretation of the answer. The main difficulty would be to do the leftover part of the integral. But again, here we can identify phases. We're going to take f of k to be localized and to be real. So there is no phase associated with it, and there is no phases associated with these quantities either, so the phases are up there.

So let's take, for example, psi incident. What is this stationary phase condition? Would be that the derivative with respect to k that we are integrating of the phase, kx minus Et over h bar must be evaluated at k naught and must be equal to 0. So that's our stationary phase approximation for the top interval.

Now remember that E is equal to h squared k squared over 2m. So what does this give you? That the peak of the pulse of the wave packet is localized at the place where the following condition holds. x minus de dk, with an h bar will give an h k t over m evaluated at k naught equals 0. So this will be x equals h bar k naught over m t. That's where the incident wave is propagating.

Now, look at that incident wave. What does it do for negative time? As time is infinite and negative, x is negative, and it's far away. Yes, the packet is very much to the left of the barrier at time equals minus infinity. And that's consistent because psi incident is only defined for x less than 0. It's only defined there.

So as long as t is negative, yes, the center of the packet is moving in. I'll maybe draw it here. The center of the packet is moving in from minus infinity into the wall, and that is the picture. The packet is here, and it's moving like that, and that's t negative. The psi incident is coming from the left into the barrier, and that's OK.

But then what happens with psi incident as t is positive? As t is positive, psi incident, well, it's just another integral. You might do it and see what you get, but we can see what we will get, roughly. When t is positive, the answer would be you get something if you have positive x. But psi incident is only for negative x.

So for negative x, you cannot satisfy the stationarity condition, and therefore, for negative x and positive time, t positive, psi incident is nothing. It's a little wiggle. There's probably something, a little bit-- look at it with Mathematica-- there will be something. But for positive t, since you only look at negative x, you don't satisfy stationarity, so you're not going to get much.

So that's interesting. Somehow automatically psi incident just exists for negative time. For time near 0 is very interesting because somehow stationary [INAUDIBLE], when you assess, you still get something, but you're going to see what the packet does as it hits the thing.

Let's do the second one of psi reflected. d dk this time would be kx with a different sign, minus kx minus E t over h bar at k naught equals 0. For the reflected wave, the phase is really the same. Yeah, this factor is a little more complicated, but it doesn't have any phase in it. It's real, so [INAUDIBLE].

So I just change a sign, so this time I'm going to get the change of sign. x is equal to minus h bar k naught over m t. And this says that for t positive, you get things. And in fact, as t is positive your are at x negative. And remember psi reflected is only defined for x negative, so you can satisfy stationary, and you're going to get something.

So for t positive, you're going to get as t increases, a thing that goes more and more to the left as you would expect. So you will get psi reflected going to the left.

I will leave for you to do the psi transmitted. It's a little different because you have now k bar, and you have to take the derivative of k bar with respect to k. It's going to be a little more interesting example.

But the answer is that this one moves as x equals h bar k bar over m t. k bar is really the momentum on the right, and since psi transmitted exists only for positive x, this relation can be satisfied for positive t. For positive t, there will be a psi transmitted.

The psi transmitted certainly exists for negative t, but for negative t, stationarity would want x to be negative, but that's not defined. So for negative t on the right, yes, psi transmitted maybe it's a little bit of something especially for times that are not too negative.

But the picture is that stationary phase tells you that these packets, psi incident, pretty much exist just for negative t and psi reflected and psi transmitted exist for positive t. And these are consequences of the fact that psi incident and psi reflected exist for negative x. The other exists for positive x, and that coupled with stationarity produces the physical picture that you expect intuitively, that the incident wave is just something, part of the solution that exists just at the beginning. And somehow it whistles away. Some of it becomes transmitted, some of it becomes reflected.