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Now that we've introduced h , h is a very important quantity in quantum mechanics. So let's talk a little more about h , its units, and we already put one number that I really wish you will remember. Now let's talk about the units of h and some other things you can do with h . So units of h .

ZWIEBACH:

So if you have a quantity that appears for the first time and as it appears here, $E = h\nu$, this is a good place to understand the units of h because the units of h would be units of energy divided by units of frequency. And I put this square brackets to denote units.

Now what are the units of energy? We're going to work with units that are characterized by M , L and T -- mass, length, and time. So energy, you think kinetic energy and you say, MV^2 , so that's a mass and velocity squared is L^2/T^2 . So that's units of energy. Frequency is cycles per unit time. Cycles have a number of units, so it's $1/T$ here. So then you say that it's ML^2/T . So that's the first answer and that's a nice answer, although it's never quite that useful in this way, so we try to rearrange it.

And I will rearrange it the following way to think-- you see, it's nice to think of what physical quantity that we are familiar, has units of \hbar . We know these units of \hbar are energy over frequency, but that's not a single physical quantity, so let's look at it and separate this as L times MLT . That's the same thing. And then I see an interesting thing-- this is the units of position, or length. Length or radius. Distance. And this has the units of momentum p . Momentum. So this product has the units of angular momentum.

And perhaps that's the most important quantity that has the units of \hbar . It's something that you should remember. Of \hbar has units of angular momentum, that's why when people talk about a particle of spin $1/2$, they say the angular momentum is $1/2$ of \hbar , and that has the right units. So spin $1/2$ particle-- $1/2$ particle-- means that the magnitude of the intrinsic angular momentum is $1/2$ of \hbar . h or \hbar have the same units, they just differ by a 2π that-- unfortunately, we have to be careful about that 2π , it affects numbers and some formulas are nicer without the bar, some formulas are less nice.

So OK. So another thing that you could say is that this h allows you to construct all kinds of new quantities. And that's a nice thing to do. Whenever you have a new constant of nature that comes up, and we have the speed of light, Planck's constant, Newton's constant-- seem

to be the three fundamental units of nature-- you can do some things. And you can look at this quantity-- h is proportional to rp and get an inspiration. So you can think h has units of r times p . And you can say, look-- if I have any particle with mass M , I can now associate a length to it. I can invent a length associated to that particle.

And how do I do it? Well, this has units of length, so all I have to do is take h and divide by p . Well, that will be one way to get the length where p is the momentum, and it will be called the de Broglie wavelength.

But there is another way. Suppose this particle is just not moving and you have the momentum and you say, wow, momentum is not moving, so what's going on here? So think of it at rest and then you say, well, you still can construct a length. You can put h and divide by the mass times the velocity of light, why not? That's a velocity, it is a constant of nature. So that way, you associate a length to any particle of a given mass. You don't have to tell me what is the momentum. You can just know the mass and it has a length associated to it.

So it's called the Compton-- Compton-- wavelength of a particle. And I want to make sure you don't confuse, it's not the same as de Broglie wavelength that we will see later. It's not the same as the de Broglie wavelength. This is the Compton wavelength of the particle. And you can say, all right, good, you give me a particle of some mass, I can tell you what a length associated to it-- why would it be important? It will be important in two different ways-- through an experiment and through a thought experiment, which I want to do right now.

You see, I could ask the following question-- I have this particle, has a mass M . I use the speed of light, so with that mass M , I could associate out to this particle a rest energy. MC^2 squared. That's the rest energy. And then I could ask, what is the wavelength of a photon that has the same energy as the rest energy of this particle? So you translate the question into a question of a length. Once you have some energy, there is a natural length, which is the wavelength of a photon with that energy.

So let's ask this question independently of what we did. So what is the wavelength-- wavelength-- of a photon whose energy is the rest mass-- rest mass-- of a particle? So the rest mass is MC^2 squared, and that's the energy of this photon. And we know that energy of a photon $h\nu$ or hC over λ , and there, we can calculate the λ . λ is hC over MC^2 squared, and no surprise, it gives us h over MC and that thing is the Compton wavelength. So it's sometimes called Compton of the particle of mass M .

So this is a way that you can think of this particle. You think of a particle, you have a Compton wavelength, and that Compton wavelength is the wavelength of light that has that rest energy. And that actually has experimental implications in high energy particle physics. Because if you have an electron and it has a Compton wavelength, and you shine a photon that has that size, that photon is carrying as much energy as the rest energy of the electron. And in particle theory and quantum field theory, particles can be created and destroyed, so this photon maybe can do some things and create more particles out of this electron, particle equation could start. happening.

So it will be difficult to isolate a particle to a size smaller than its Compton wavelength, because the photons could do such damage to the particle by creating new particles or doing other things to it.

So for an electron, let's calculate the Compton wavelength. So Compton of an electron would be $\frac{h}{m_e c}$, and you would do $2\pi \frac{\hbar}{m_e c}$ and $m_e c$ squared. And you've got $2\pi \cdot 197.33 \text{ MeV fermi}$, and here you would have 0.511 MeV . So this gives you $2,426 \text{ fermi}$, or about 2.426 picometers . Picometers is kind of a natural length. Picometer is 10^{-12} meters. The Bohr radius is about 50 picometers , so that's how big this thing is.

Is it still much bigger than the size of the nucleus? The nucleus is a few fermis. A single proton is about a fermi big. And nucleus grow slowly, so you can have a big nucleus with 200 particles maybe of a radius of 7 or 8 fermi. So this is still a lot bigger and it's a very interesting quantity that will show up very soon.