Welcome back to 8.033!

Richard Feynman, 1918-1988

Summary of last lecture:

- Atomic physics

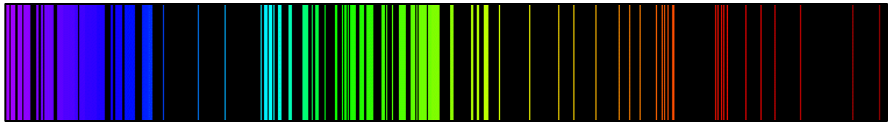


Image courtesy of Wikipedia.

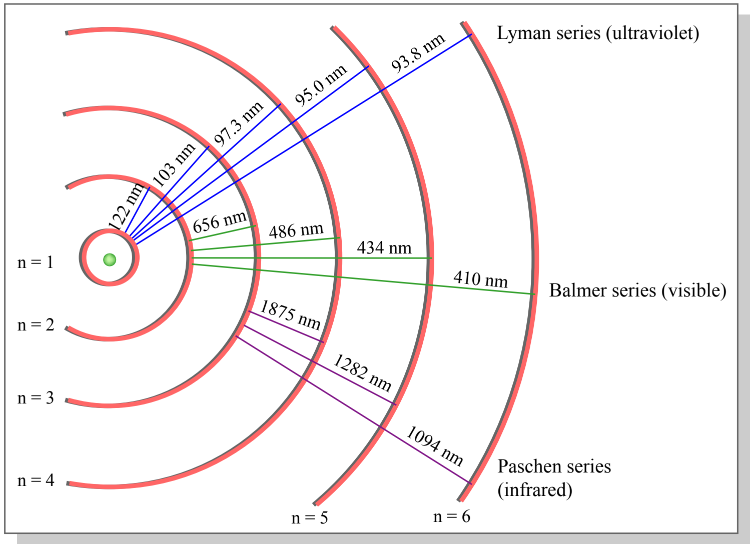


Figure by MIT OCW.

- Nuclear physics

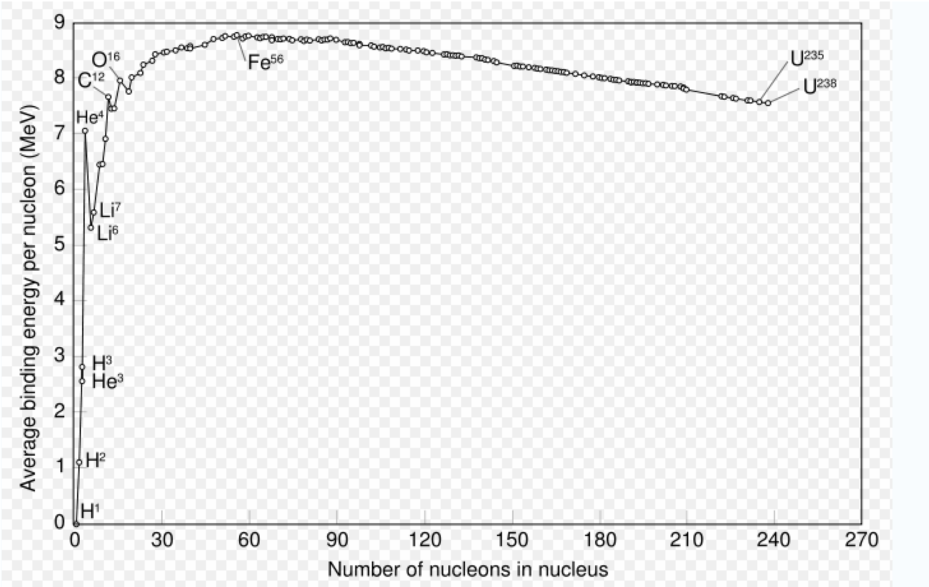


Image courtesy of Wikipedia.

Working with photons:

Image courtesy of Wikipedia.

- Photon 4-vector:

$$\mathbf{P} = \hbar \begin{pmatrix} \mathbf{k} \\ k \end{pmatrix},$$

  where $k = \omega/c$.

- So $p = E/c$ for photons.

- Comparing $\mathbf{P}$ with the wave 4-vector $\mathbf{K}$ shows that

$$\mathbf{P} = \hbar \mathbf{K}.$$

  This relation in fact holds for *all* particles, even massive ones — as you'll see when you get to wave-particle duality in quantum mechanics. If you take a field theory course, you'll see this pop right out of the so-called Klein-Gordon equation.

- Doppler effect is just special case of $\mathbf{P}$-transformation for zero rest mass — show on PS6.

MIT Course 8.033, Fall 2006, Lecture 13

Max Tegmark

Today's topics:

- Particle physics
- The greatest unsolved problems in physics

Particle physics

CERN particle accelerator

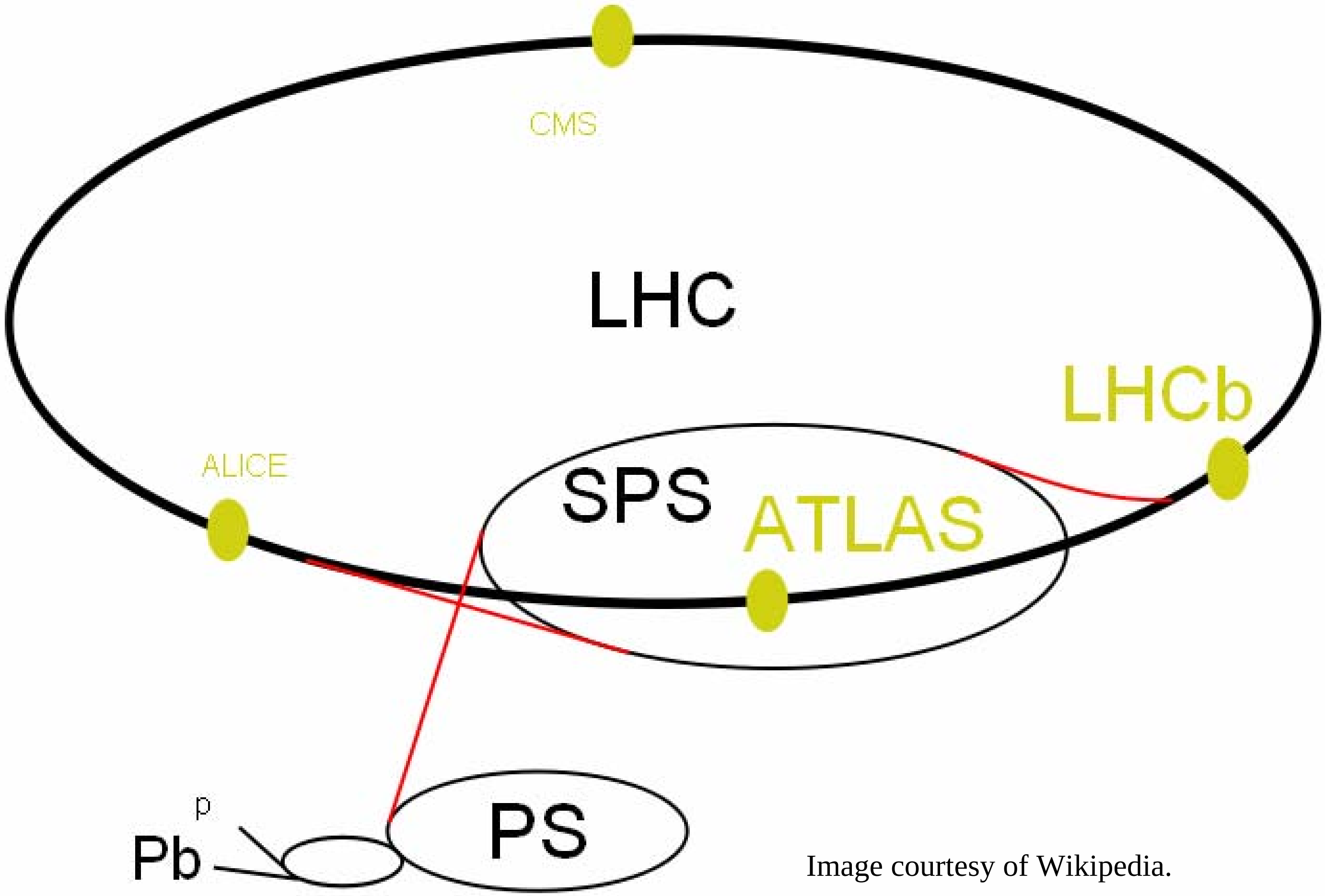


Image courtesy of Wikipedia.

Linear particle accelerator (Fermilab)

Image courtesy of Wikipedia.

How derive curvature radius of particle tracks?

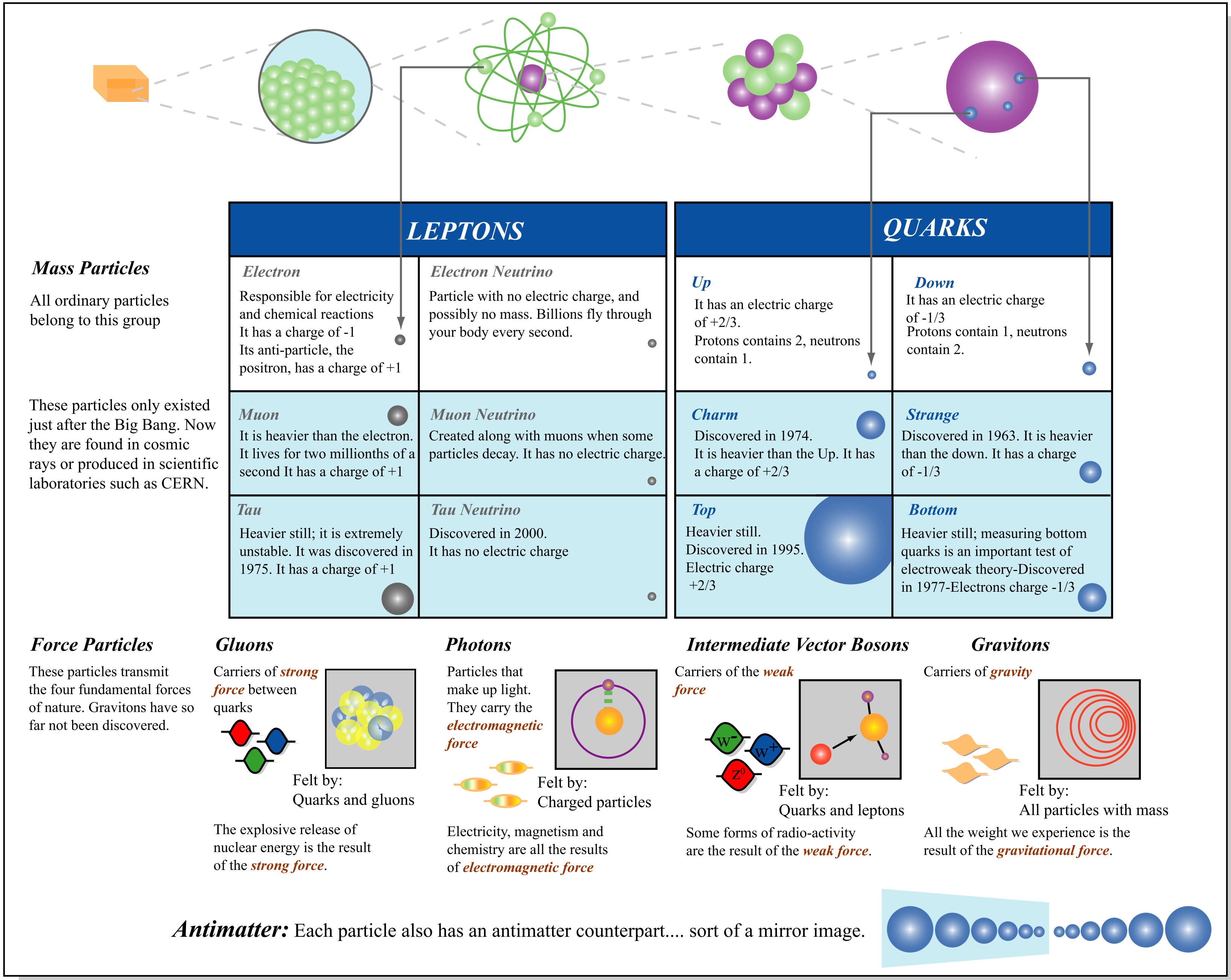


Figure by MIT OCW.

Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

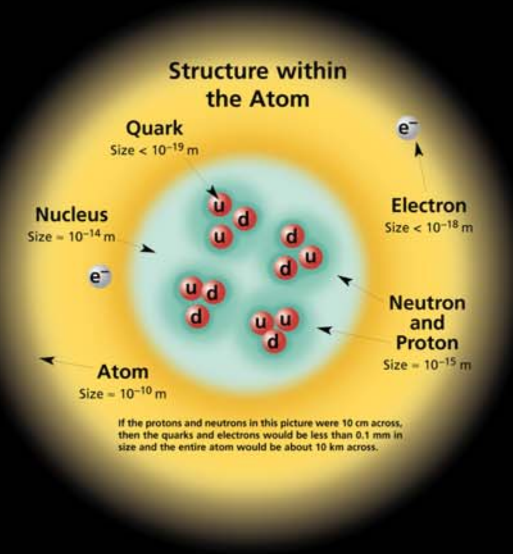
Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $= 1.05\times10^{-34}$ J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV $= 10^9$ eV $= 1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c^2 $= 1.67\times10^{-27}$ kg.



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons. There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$

Mesons are bosonic hadrons. There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

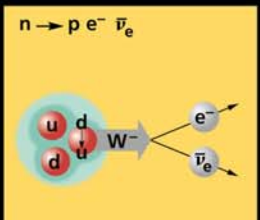
Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures

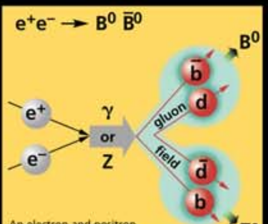
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

$n \rightarrow p\, e^-\, \bar{\nu}_e$



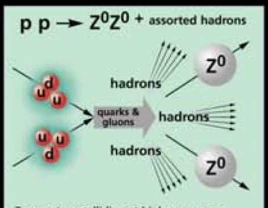
A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

$e^+e^- \rightarrow B^0\, \bar{B}^0$



An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

$p\, p \rightarrow Z^0 Z^0$ + assorted hadrons



Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at **http://ParticleAdventure.org**

This chart has been made possible by the generous support of:
U.S. Department of Energy
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Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
BURLE INDUSTRIES, INC.

http://CPEPweb.org

Rest energies of common particles

Particle	Symbol	Rest energy
electron	e^-	0.511 MeV
muon	μ^-	105.6 MeV
tau	τ^-	1777 MeV
proton	p^+	938.26 MeV
neutron	n	939.55 MeV
charged pion	$\pi^+,\ \pi^-$	139.6 MeV
neutral pion	π^0	135.0 MeV
neutrinos	$\nu_e,\ \nu_\mu,\ \mu_\tau$	< 0.14 eV
photon	γ	0 MeV
graviton	g	0 MeV
$C^{12}/12$	amu	931.5 MeV

WHY?

- Open question: why?
  Why is proton/electron mass ratio 1836, say?

The Standard Model Lagrangian

$$\mathcal{L} =$$

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\frac{1}{2}ig_s^2(\bar q^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar G^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$
$$M^2 W^+_\mu W^-_\mu - \frac12 \partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac12\partial_\mu A_\nu \partial_\mu A_\nu - \frac12 \partial_\mu H\partial_\mu H -$$
$$\frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} +$$
$$\frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\frac18 g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$
$$gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac14 g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\frac14 g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac12 g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac12 ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \bar d^\lambda_j(\gamma\partial +$$
$$m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar e^\lambda\gamma e^\lambda) + \frac23(\bar u^\lambda_j\gamma u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma d^\lambda_j)] + \frac{ig}{4c_w}Z^0_\mu[(\bar\nu^\lambda\gamma^\mu(1+$$
$$\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\frac43 s_w^2 - 1 - \gamma^5)u^\lambda_j) +$$
$$(\bar d^\lambda_j\gamma^\mu(1 - \frac83 s_w^2 - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(1+$$
$$\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] +$$
$$\frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) +$$
$$i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+$$
$$\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] -$$
$$\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac g2\frac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) +$$
$$\bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2 - M^2)X^- + \bar X^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y +$$
$$igc_wW^+_\mu(\partial_\mu\bar X^0 X^- - \partial_\mu\bar X^+ X^0) + igs_wW^+_\mu(\partial_\mu\bar Y X^- - \partial_\mu\bar X^+ Y) +$$
$$igc_wW^-_\mu(\partial_\mu\bar X^- X^0 - \partial_\mu\bar X^0 X^+) + igs_wW^-_\mu(\partial_\mu\bar X^- Y - \partial_\mu\bar Y X^+) +$$
$$igc_wZ^0_\mu(\partial_\mu\bar X^+ X^+ - \partial_\mu\bar X^- X^-) + igs_wA_\mu(\partial_\mu\bar X^+ X^+ - \partial_\mu\bar X^- X^-) -$$
$$\frac12 gM[\bar X^+ X^+ H + \bar X^- X^- H + \frac{1}{c_w^2}\bar X^0 X^0 H] + \frac{1-2c_w^2}{2c_w}igM[\bar X^+ X^0\phi^+ -$$
$$\bar X^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0 X^-\phi^+ - \bar X^0 X^+\phi^-] + igMs_w[\bar X^0 X^-\phi^+ -$$
$$\bar X^0 X^+\phi^-] + \frac12 igM[\bar X^+ X^+\phi^0 - \bar X^- X^-\phi^0]$$

(From T.D. Gutierrez)

Standard model parameters:

Parameter	Meaning	Measured value
g	Weak coupling constant at M_Z	0.6425
θ_W	Weinberg angle	0.4908
g_s	Strong coupling constant	≈ 1.2
μ^2	Quadratic Higgs coefficient	$\sim -10^{-33}$
λ	Quartic Higgs coefficient	$\sim 1?$
G_e	Electron Yukawa coupling	2.94×10^{-6}
G_μ	Muon Yukawa coupling	0.000607
G_τ	Tauon Yukawa coupling	0.0102156233
G_u	Up quark Yukawa coupling	0.000016 ± 0.000007
G_d	Down quark Yukawa coupling	0.00003 ± 0.00002
G_c	Charm quark Yukawa coupling	0.0072 ± 0.0006
G_s	Strange quark Yukawa coupling	0.0006 ± 0.0002
G_t	Top quark Yukawa coupling	1.002 ± 0.029
G_b	Bottom quark Yukawa coupling	0.026 ± 0.003
$\sin\theta_{12}$	Quark CKM matrix angle	0.2243 ± 0.0016
$\sin\theta_{23}$	Quark CKM matrix angle	0.0413 ± 0.0015
$\sin\theta_{13}$	Quark CKM matrix angle	0.0037 ± 0.0005
δ_{13}	Quark CKM matrix phase	1.05 ± 0.24
θ_{qcd}	CP-violating QCD vacuum phase	$< 10^{-9}$
G_{ν_e}	Electron neutrino Yukawa coupling	$< 1.7 \times 10^{-11}$
G_{ν_μ}	Muon neutrino Yukawa coupling	$< 1.1 \times 10^{-6}$
G_{ν_τ}	Tau neutrino Yukawa coupling	< 0.10
...	Neutrino mixing parameters	
ρ_Λ	Dark energy density	$(9.3 \pm 2.5) \times 10^{-124}$
ξ_b	Baryon mass per photon ρ_b/n_γ	$(0.49 \pm 0.03) \times 10^{-28}$
ξ_c	Cold dark matter mass per photon ρ_c/n_γ	$(2.7 \pm 0.2) \times 10^{-28}$
ξ_ν	Neutrino mass per photon $\rho_\nu/n_\gamma = \frac{3}{11}\sum m_{\nu_i}$	$< 0.9 \times 10^{-28}$
Q	Scalar fluctuation amplitude δ_H on horizon	$(2.0 \pm 0.2) \times 10^{-5}$
n_s	Scalar spectral index	0.98 ± 0.02
α_n	Running of spectral index $dn_s/d\ln k$	$\lvert\alpha\rvert \lesssim 0.01$
r	Tensor-to-scalar ratio $(Q_t/Q)^2$	$\lesssim 0.36$
n_t	Tensor spectral index	Unconstrained
Ω_{tot}	Spatial curvature	1.01 ± 0.02
w	Dark energy equation of state	-1 ± 0.1

$C = \hbar = G = k_b = q_e = 1$

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$\Omega_{\rm tot}$	Spatial curvature	1.01 ± 0.02
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How measure?

Why these values?

Table 2: Derived physical parameters, in extended Planck units $c = \hbar = G = k_b = q_e = 1$.

Parameter	Meaning	Definition	Measured value
ϵ	Electromagnetic coupling constant	$g \sin\theta_W$	≈ 0.302822113
α	Electromagnetic integaction strength	$\epsilon^2/4\pi = g^2 \sin^2\theta_W/4\pi$	$1/137.03599911(46)$
α_w	Weak interaction strength	$g^2/4\pi$	≈ 0.02
α_s	Strong interaction strength	$g_s^2/4\pi$	≈ 0.12
α_g	Gravitational coupling constant	$Gm_\mathrm{p}^2/\hbar c = m_\mathrm{p}^2$	$\approx 5.9046 \times 10^{-39}$
m_W	$W^\pm$ mass	$vg/2$	(80.425 ± 0.038) GeV
m_Z	Z mass	$vg/2\cos\theta_W$	(91.1876 ± 0.0021) GeV
G_F	Fermi constant	$1/\sqrt{2}v^2$	$\approx 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$
m_H	Higgs mass	$\sqrt{-\mu^2/2}$	100-250 GeV?
v	Higgs vacuum expectation value	$\sqrt{-\mu^2/\lambda}$	246 GeV
m_e	Electron mass	$vG_e/\sqrt{2}$	(510998.92 ± 0.04)eV
m_μ	Muon mass	$vG_\mu/\sqrt{2}$	(105658369 ± 9)eV
m_τ	Tauon mass	$vG_\tau/\sqrt{2}$	(1776.99 ± 0.29) MeV
m_u	Up quark mass	$vG_u/\sqrt{2}$	$(1.5 - 4)$ MeV
m_d	Down quark mass	$vG_g/\sqrt{2}$	$(4 - 8)$ MeV
m_c	Charm quark mass	$vG_c/\sqrt{2}$	$(1.15 - 1.35)$ GeV
m_s	Strange quark mass	$vG_s/\sqrt{2}$	$(80 - 130)$ MeV
m_t	Top quark mass	$vG_t/\sqrt{2}$	(174.3 ± 5.1) GeV
m_b	Bottom quark mass	$vG_b/\sqrt{2}$	$(4.1 - 4.9)$ GeV
m_{ν_e}	Electron neutrino mass	$vG_{\nu_e}/\sqrt{2}$	< 3eV
m_{ν_μ}	Muon neutrino mass	$vG_{\nu_\mu}/\sqrt{2}$	< 0.19 MeV
m_{ν_τ}	Tau neutrino mass	$vG_{\nu_\tau}/\sqrt{2}$	< 18.2 GeV
m_p	Proton mass	$2m_u + m_d$+QCD+QED	(938.27203 ± 0.00008) MeV
m_n	Neutron mass	$2m_d + m_u$+QCD+QED	(939.56536 ± 0.00008) MeV
β	Electron/proton mass ratio	m_e/m_p	$1/1836.15$
β_n	Neutron/proton mass ratio	m_n/m_p	1.001378298
Ry	Hydrogen binding energy (Rydberg)	$\mathrm{Ry} = \mathrm{m_e c^2}\alpha^2/2 = \alpha^2\alpha_\mathrm{g}^{1/2}\beta/2$	≈ 13.6057eV
a_B	Bohr radius	$a_\mathrm{B} = \hbar/cm_e\alpha = (\alpha\beta)^{-1}\alpha_g^{-1/2}$	$\approx 5.29177 \times 10^{-11}\,\mathrm{m}$
σ_t	Thomson cross section	$\frac{8\pi}{3}\left(\frac{\hbar\alpha}{m_e c}\right)^2 = \frac{8\pi}{3}\alpha^2\alpha_\mathrm{g}^{-1}\beta^{-2}$	$\approx 6.65246 \times 10^{-29}\,\mathrm{m}^2$
k_c	Coulomb's constant	$1/4\pi\epsilon_0 = \hbar c\alpha/q_e^2 = \alpha$	$1/137.03599911(46)$
η	Baryon/photon ratio	$n_\mathrm{b}/n_\gamma = \xi_\mathrm{b}/m_p$	$(6.3 \pm 0.3) \times 10^{-10}$
ξ	Matter per photon	$\xi_\mathrm{b} + \xi_\mathrm{c} + \xi_\nu = \rho_\mathrm{m}/n_\gamma = m_\mathrm{p}\eta(1 + R_\mathrm{c} + R_\nu)$	$(3.3 \pm 0.3) \times 10^{-28} \approx 4$eV
R_c	CDM/baryon density ratio	$\rho_\mathrm{c}/\rho_\mathrm{b} = \xi_\mathrm{c}/\xi_\mathrm{b} = \omega_\mathrm{cdm}/\omega_\mathrm{b}$	≈ 6
R_ν	Neutrino/baryon density ratio	$\rho_\nu/\rho_\mathrm{b} = \frac{f_\nu\omega_\mathrm{m}}{\omega_\mathrm{b}} = \frac{3}{11}\frac{M_\nu}{\eta m_\mathrm{p}}$	$\lesssim 1$
R_k	Dimensionless curvature parameter	$a^2 t$ for $a \ll a_\mathrm{eq}$	$\lesssim \times 10^{-59}$
M_ν	Sum of neutrino masses	$M_\nu = 3\rho_\nu/n_\nu = (11/3)m_\mathrm{p}\eta R_\nu$	
f_ν	Neutrino density fraction	$f_\nu = \rho_\nu/\rho_\mathrm{m} = \left[1 + \frac{11(\xi_\mathrm{b}+\xi_\mathrm{d})}{3M_\nu}\right]^{-1}$	<?
T_eq	Matter-radiation equality temperature	$\frac{30\zeta(3)}{\pi^4}\left[1 + \frac{21}{8}\left(\frac{4}{11}\right)^{4/3}\right]^{-1}\xi \approx 0.220189\xi$	$\approx 9.4 \times 10^3$K
$\rho_\mathrm{m}^\mathrm{eq}$	Matter density at equality	$\rho_\mathrm{m}(T_\mathrm{eq}) = \frac{765314352000\zeta(3)^4}{\left(242+21\times 22^{2/3}\right)^3\pi^{14}}\xi^4 \approx 0.00260042\xi^4$	$\approx ?? \times 10^{??}$
A_Λ	Dark energy domination epoch	$x_\mathrm{eq}^{1/3} = (\rho_\mathrm{m}^\mathrm{eq}/\rho_\Lambda)^{1/3} \approx 0.137514\xi^{4/3}\rho_\Lambda^{-1/3}$	3215 ± 639
ξ_wimp	WIMP dark matter density per photon	$\sim -\mu^2/\lambda g^2$	$\approx 3 \times 10^{-28}$?

Table 3: Derived physical variables, in extended Planck units $c = \hbar = G = k_b = q_e = 1$.

Parameter	Meaning	Definition	Measured value
T	CMB temperature	(Acts as a time variable)	2.725K (today)
n_γ	Photon number density	$\frac{2\zeta(3)}{\pi^2}T^3, \quad \zeta(3) \approx 1.20206$	$0.243588T^3$
n_ν	Neutrino number density	$\frac{9}{11}n_\gamma = \frac{18\zeta(3)}{11\pi^2}T^3$	$0.199299T^3$
ρ_γ	Photon density	$\frac{\pi^2}{15}T^4$	$0.657974T^4$
$\rho_{\rm b}$	Baryon density	$\xi_{\rm b}n_\gamma = m_p\eta n_\gamma = \frac{2\zeta(3)}{\pi^2}\xi_{\rm b}T^3$	$0.243588\xi_{\rm b}T^3$
$\rho_{\rm c}$	CDM density	$\xi_{\rm c}n_\gamma = R_{\rm c}\rho_{\rm b} = \frac{2\zeta(3)}{\pi^2}\xi_{\rm c}T^3$	$0.243588\xi_{\rm c}T^3$
ρ_ν	Neutrino density (massive)	$\xi_\nu n_\gamma = R_\nu\rho_{\rm b} = \frac{2\zeta(3)}{\pi^2}\xi_\nu T^3 = \frac{n_\nu M_\nu}{3} = \frac{3}{11}n_\gamma M_\nu$	$0.243588\xi_\nu T^3$
ρ_ν^γ	Neutrino density (massless)	$\frac{21}{8}\left(\frac{4}{11}\right)^{4/3}\rho_\gamma \approx 0.681322\rho_\gamma$	$0.448292T^4$
$\rho_{\rm m}$	Total matter density	$\rho_{\rm b}+\rho_{\rm c}+\rho_\nu = n_\gamma\xi = \frac{2\zeta(3)}{\pi^2}(\xi_{\rm b}+\xi_{\rm c}+\xi_\nu)T^3 = \omega_\Lambda \mathbf{x}$	$0.243588\xi T^3$
$\rho_{\rm k}$	Curvature density	$\pm\frac{3}{8\pi Ga^2}$	
A	Expansion factor since equality	$a/a_{\rm eq} = A_\Lambda x^{1/3} \approx 0.137514\xi^{4/3}\rho_{\rm m}^{-1/3}$	$\sim 3.5\times10^3$ today
x	Dark energy/matter ratio	$\rho_\Lambda/\rho_{\rm m} = (A/A_\Lambda)^3 = \frac{\pi^2}{2\zeta(3)}\frac{\rho_\Lambda}{\xi T^3}$	$\sim 7/3$ today
H_*	Hubble reference rate	$H/h = 100{\rm km\,s^{-1}Mpc^{-1}}$	$(9.7779{\rm Gyr})^{-1}$
ρ_h	Hubble reference density	$3(H/h)^2/8\pi G = 3(100{\rm km\,s^{-1}Mpc^{-1}})^2/8\pi G$	$1.87882\times10^{-26}{\rm kg/m^3}$
$\omega_{\rm b}$	Baryon density parameter	$\Omega_{\rm b}h^2 = \rho_{\rm b}/\rho_h = \xi_{\rm b}n_\gamma/\rho_h = (2\zeta(3)/\pi^2)\xi_{\rm b}T^3$	0.023 ± 0.001 (today)
$\omega_{\rm cdm}$	Cold dark matter density parameter	$\Omega_{\rm cdm}h^2 = \rho_{\rm c}/\rho_h = \xi_{\rm c}n_\gamma/\rho_h = (2\zeta(3)/\pi^2\rho_h)\xi_{\rm c}T^3$	0.1?? (today)
ω_ν	Neutrino density parameter	$\Omega_\nu h^2 = \rho_\nu/\rho_h = \xi_\nu n_\gamma/\rho_h = (2\zeta(3)/\pi^2\rho_h)\xi_\nu T^3$	<??? (today)
$\omega_{\rm d}$	Dark matter density parameter	$\Omega_{\rm d}h^2 = \omega_{\rm cdm}+\omega_\nu = (2\zeta(3)/\pi^2\rho_h)(\xi_{\rm b}+\xi_{\rm c})T^3$	$\approx 0.023\pm0.001$ (today)
$\omega_{\rm m}$	Matter density paramter	$\Omega_{\rm m}h^2 = \omega_{\rm b}+\omega_{\rm cdm}+\omega_\nu = (2\zeta(3)/\pi^2\rho_h)(\xi_{\rm b}+\xi_{\rm c}+\xi_\nu)T^3$	0.1?? (today)
ω_Λ	Dark energy density paramter	$\omega_\Lambda = \Omega_\Lambda h^2 = \rho_\Lambda/\rho_h$	0.26 ± 0.07
ω_γ	Photon density parameter	$\rho_\gamma/\rho_h = (\pi^2/15\rho_h)T^4 \approx 1.80618\times10^{122}T^4$	0.0000247 ± 0.0000004 (today)
H	Hubble parameter	$\left[\frac{8\pi G}{3}(\rho_\Lambda+\rho_{\rm m}+\rho_\gamma+\rho_{\rm k})\right]^{1/2}$	$\approx (10{\rm Gyr})^{-1}$ today
h	Dimensionless Hubble parameter	$h = H/H_*$	≈ 0.7 today
t	Age of Universe	$\int_0^a H(a')^{-1}d\ln a'$	≈ 14Gyr today

Particle physics processes

- We know of four fundamental interactions: gravitational, electromagnetic, weak and strong. In particle physics, the first is negligible.

- See the handouts for summmaries of particles and interactions.

- Summary of particle physics processes we consider:

  - Absorption (two particles in, one out)
  - Emission/decay (one particle in, two out)
  - Collision/scattering/annihilation/creation (two particles in, two out)

- **Footnote:** if you take a course in quantum field theory, you'll find that two in, two out ("four-vertex") interactions can generally be reduced to two separate three-vertex interactions, where the momentum and energy transfer between the two colliding particles is mediated by an intermediate particle. For instance, an elastic collision between two electrons can be reduced to a photon exchange: one electron emits a photon that's later absorbed by the other.

- Which processes are allowed in nature? All that aren't forbidden by a conservation law, *e.g.*,
  - Energy-momentum conservation ($\mathbf{P}$ conserved)
  - Charge conservation
  - Baryon number conservation
  - Lepton number conservation
  - Parity conservation (except in weak interactions)

- Everything is provisional:

  - Momentum conservation appeared to be violated in β-decay, but was rescued with neutrino discovery (proposed by Wolfgang Pauli 1931, detected by Fred Reines & Clyde Cowan 1956).
  - Parity conservation was believed to be universally valid until the shock of 1956 (Yang, Lee, Wu).
  - Many physicists believe (but haven't shown) that lepton and/or baryon number is violated ever so slightly, *e.g.*, that protons decay if you wait $\gg 10^{32}$ years.

- There's more to it: computing lifetimes and scattering probabilities requires quantum field theory - in this course, we'll limit ourselves to drawing conclusions from energy-momentum conservation.

Less energy

Common interaction processes

- Chemical reactions: atoms get rearranged in new ways, perhaps emitting or absorbing photons and electrons. Non-relativistic.
- Nuclear reactions: nucleons get rearranged in new ways, perhaps emitting or absorbing photons, electrons, positrons and neutrinos (electron/positrons and neutrinos must be involved whenever there are conversions betweens protons and neutrons, to conserve charge and lepton number).
- Elementary particle interactions: energy, momentum, charge, lepton number *etc.* gets rearranged in new ways, corresponding to scattering, destruction and creation of particles.

More energy

Examples:

- Molecule + molecule $\rightarrow$ new molecules + γ (chemical reaction)
- γ + atom $\rightarrow$ exited atom (excitation)
- γ + atom $\rightarrow$ e^- + atom (ionization; photoelectric effect)
- Nucleus + nucleus $\rightarrow$ new nuclei + $\gamma/e^-/\nu$ (nuclear reaction)
- $n \rightarrow p^+ + e^- + \bar{\nu}$ (beta decay)
- $\gamma + \gamma \rightarrow e^- + e^+$ (pair creation)
- γ + particle $\rightarrow$ particle $+ e^- + e^+$
- $\gamma + e^- \rightarrow \gamma + e^-$ (Compton scattering)

Nuclear physics terminology

- The *atomic number* Z of a nucleus is its number of protons.
- The *atomic weight* A of a nucleus is its number of nucleons (protons + neutrons).
- Z determines the name of the element (its order in the periodic table).
- Nuclei with same Z and different A are said to be different *isotopes* of the same element.
- Notation example: Fe^{56} means $Z = 26$ (iron) and $A = 56$.
- The *mass excess* for a nucleus is $m_0 - A$ amu, *i.e.*, its rest mass minus the number of nucleons times amu.
- By this definition, the mass excess of C^{12} is zero.
- Historically (before people knew exactly what they were), Helium nuclei, electrons and energetic photons were called α-particles, β-particles and γ-particles, respectively, and linguistic vestiges of this live on:
  - The process $n \to p^+ + e^- + \bar{\nu}$ is called β-decay.
  - High energy photons are denoted γ-rays, and photons are denoted γ (which is of course confusing in 8.033)!

α β γ

Photon emission & absorbtion:

- Photon absorbtion ($X + \gamma \rightarrow X^*$): If a particle at rest with mass m_0 absorbs a photon of frequency ω, it acquires a speed

$$\beta' = \frac{\hbar\omega}{m_0 c^2 + \hbar\omega}.$$

- Photon emission with recoil ($X^* \rightarrow X + \gamma$): if a particle emits energy $\hbar\omega$ as a photon, thereby reducing its rest mass from m_0 to $m_0' \equiv m_0 - Q_0/c^2$, then

$$\hbar\omega = \left(1 - \frac{Q_0}{2m_0c^2}\right) Q_0.$$

  Thus the photon energy $\hbar\omega < Q_0$ because of recoil, whereby some of the released energy Q_0 turns into kinetic energy of the recoiling particle.

- This works in reverse too: to increase its rest energy by Q_0, the particle needs to absorb a photon with energy $\hbar\omega > Q_0$ to compensate for the recoil.

- This recoil effect (the term $Q_0/2m_0c^2$ in the parenthesis above) is normally negligibly small $\sim 10^{-8}$ for typical *atomic* transition energies ($\sim 10eV$) — for comparison, Doppler line broadening is of order $\beta \sim 10^{-6}$ for room temperature atoms moving with thermal velocities of hundreds of meters per second.

- However, it is important for *nuclear* transition energies, which are of order a thousand times larger (Moon's experiment 1951).

- Mössbauer effect (1961 Nobel Prize for Ph.D. thesis work) all but eliminates recoil, making m_0 the rest mass of the whole crystal rather than one particle. Allows measuring 2cm/s Doppler shifts!

- Pound & Rebka experiment from Harvard Tower 1960 used this to detect tiny $\sim 10^{-14}$ gravitational redshift.

Harvard Tower Experiment
(Pound & Rebka 1960)

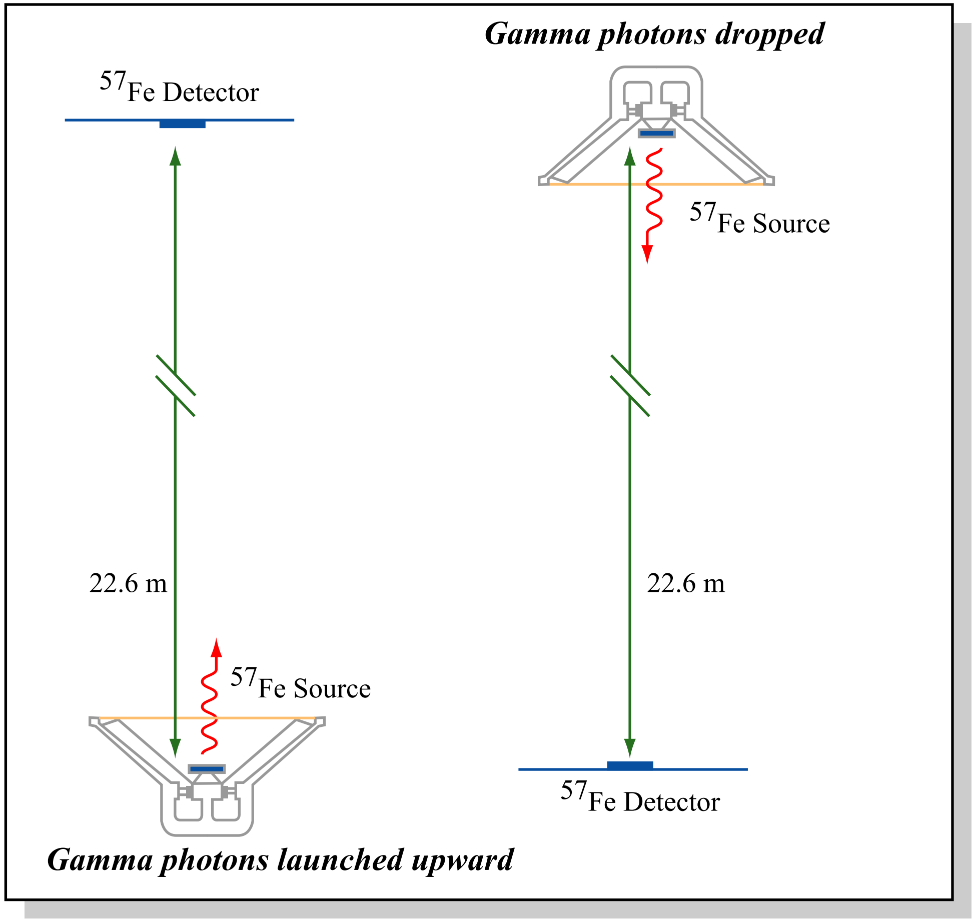
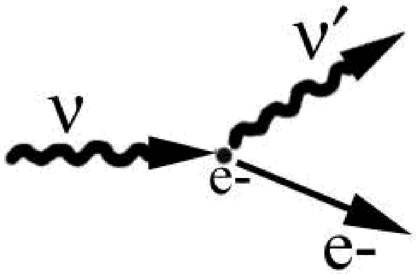


Figure by MIT OCW.

Over 22.6 meters, the gravitational redshift is only 5×10^{-15}, but the Mössbauer effect with the 14.4 keV gγ–ray from iron-57 has a high enough resolution to detect that difference.

Compton scattering

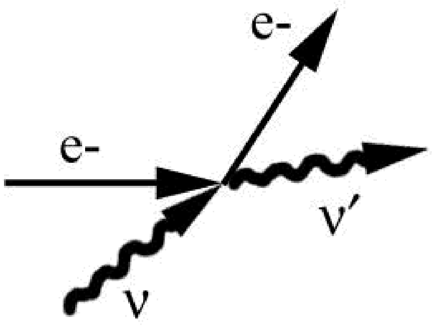


$$\nu' < \nu$$

Electron is initially at rest

e- gains energy

Inverse Compton scattering



$$\nu' > \nu$$

High energy e- initially

e- loses energy

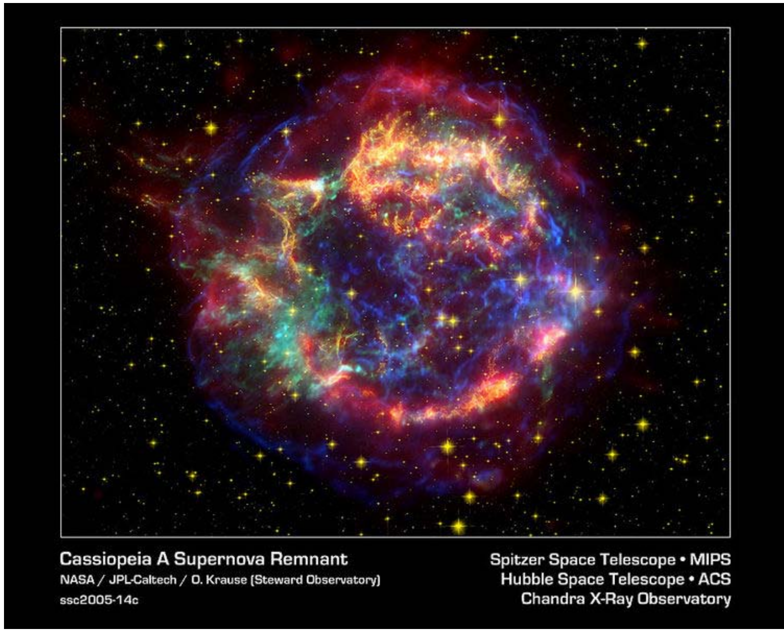


Image courtesy of NASA and the ESA.

Compton scattering

- Compton scattering ($\gamma + e^- \rightarrow \gamma + e^-$) with electron initially at rest:

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1 - \cos\theta)}$$

- Such an elastic photon-electron collision is called *Compton scattering* when the photon transfers energy to the electron and *inverse Compton scattering* when the electron transfers energy to the photon.

- The former occurs when shining x-rays at matter.

- The latter occurs frequently in astrophysics.

- Them two are of course equivalent in special relativity, since you can always Lorentz transform into a frame where, before the collision, either the electron has much more energy than the photon or vice versa.

Less energy

Common interaction processes

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- Nuclear reactions: nucleons get rearranged in new ways, perhaps emitting or absorbing photons, electrons, positrons and neutrinos (electron/positrons and neutrinos must be involved whenever there are conversions betweens protons and neutrons, to conserve charge and lepton number).

- Elementary particle interactions: energy, momentum, charge, lepton number *etc.* gets rearranged in new ways, corresponding to scattering, destruction and creation of particles.

More energy

UNSOLVED PROBLEMS

UNSOLVED PROBLEMS:

- Where do the “constants” come from? Why 3+1 dimensions?
- Is there a quantum gravity TOE? (M-theory? Black hole evaporation?)
- Proton decay?
- SUSY?
- Higgs?
- Neutrino properties?
- Dark energy?
- Dark matter?
- Inflation?

PREDICTING PARAMETERS

PREDICTING

It's tough to make predictions, especially about the future.

Yogi Berra

Parameter status?

Mass of Earth	5.9742×10^{24} kg
Semimajor axis of Earth's orbit	149,597,870,691 m
Mass of electron	$9.10938188 \times 10^{-31}$ kg
Bohr Radius of Hydrogen atom	$5.29177 x 10^{-11}$ m

Parameter status?

Mass of Earth	5.9742×10^{24} kg	← Environmental
Semimajor axis of Earth's orbit	149,597,870,691 m	← Environmental
Mass of electron	$9.10938188 \times 10^{-31}$ kg	← Fundamental?
Bohr Radius of Hydrogen atom	$5.29177 \text{x} 10^{-11}$ m	← Fundamental?

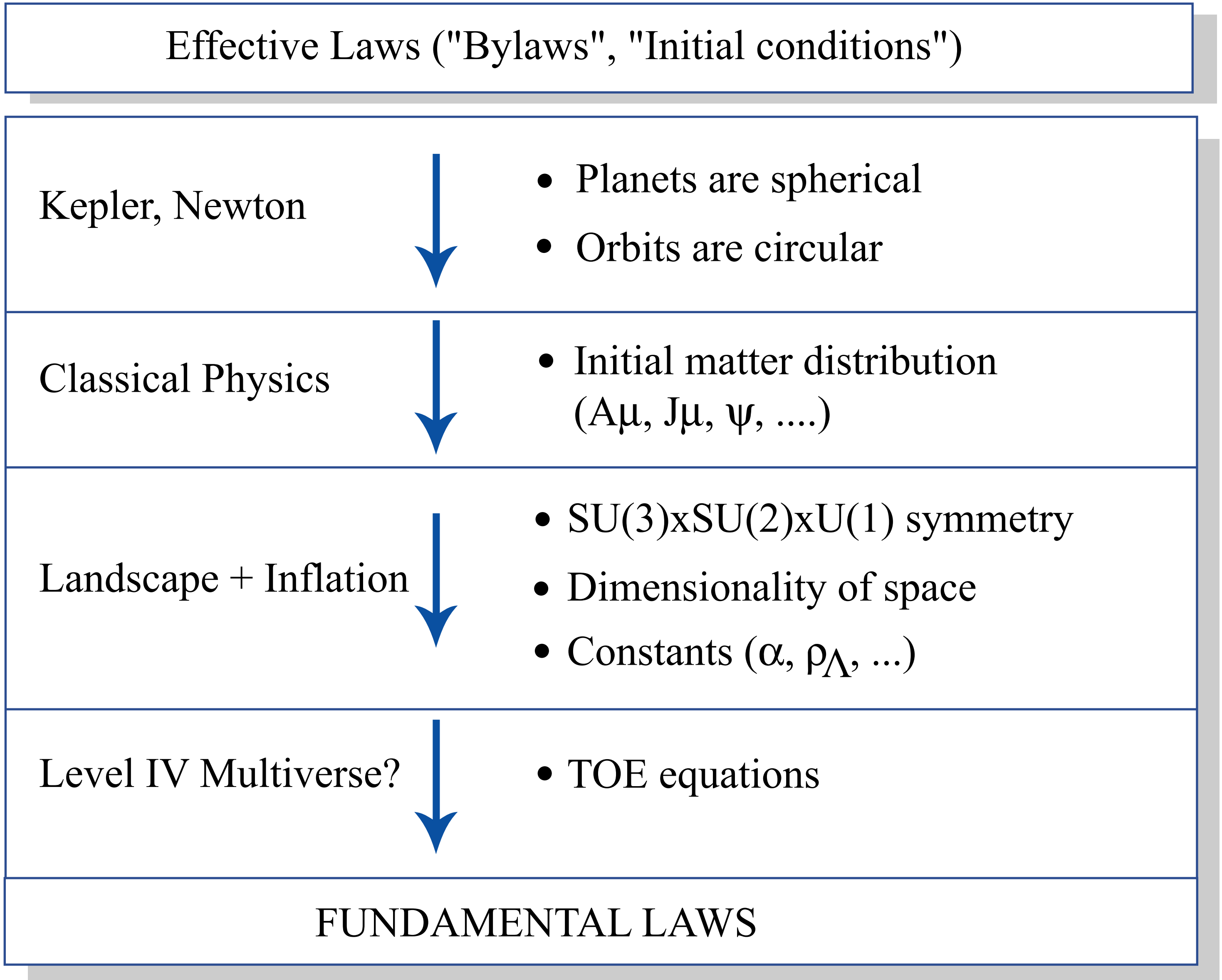


Figure by MIT OCW.

What are the 4 multiverse levels like?

1) Same effective laws of physics, different initial conditions

2) Same fundamental laws of physics, different effective laws

3) Nothing qualitatively new

4) Different fundamental laws of physics

Three little numbers:

$$\alpha \equiv e^2 = \frac{e^2}{\hbar c} = \frac{q_e^2}{4\pi\epsilon_0\hbar c} \approx 1/137.03599976$$

$$\beta \equiv \frac{m_\mathrm{e}}{m_\mathrm{p}} \approx 1/1836.15$$

$$m_\mathrm{p} = \frac{m_\mathrm{p}}{m_\mathrm{pl}} \approx 7.68417 \times 10^{-20}$$

$$c = \hbar = G = k_b = |q_e| = 1$$

Atom

$R \sim 1/m_p\alpha\beta$

$M \sim m_p$

Weisskopf 1975
Carr & Rees 1979

Asteroid

$R \sim 1/\alpha^{1/2}\beta^{1/2}m_p^{\,2}$

$M \sim \alpha^{3/2}\beta^{3/2}/m_p^{\,2}$

The Earth

$$R \sim 1/m_p^2 \alpha^{1/2} \beta$$

$$M \sim \alpha^{3/2} / m_p^2$$

Weisskopf 1975
Carr & Rees 1979

Mt. Fuji

$$R \sim 1/m_p^{3/2}\alpha^{3/4}\beta$$

$$M \sim \alpha^{3/4} / m_p^{1/2}$$

Carr & Rees 1979

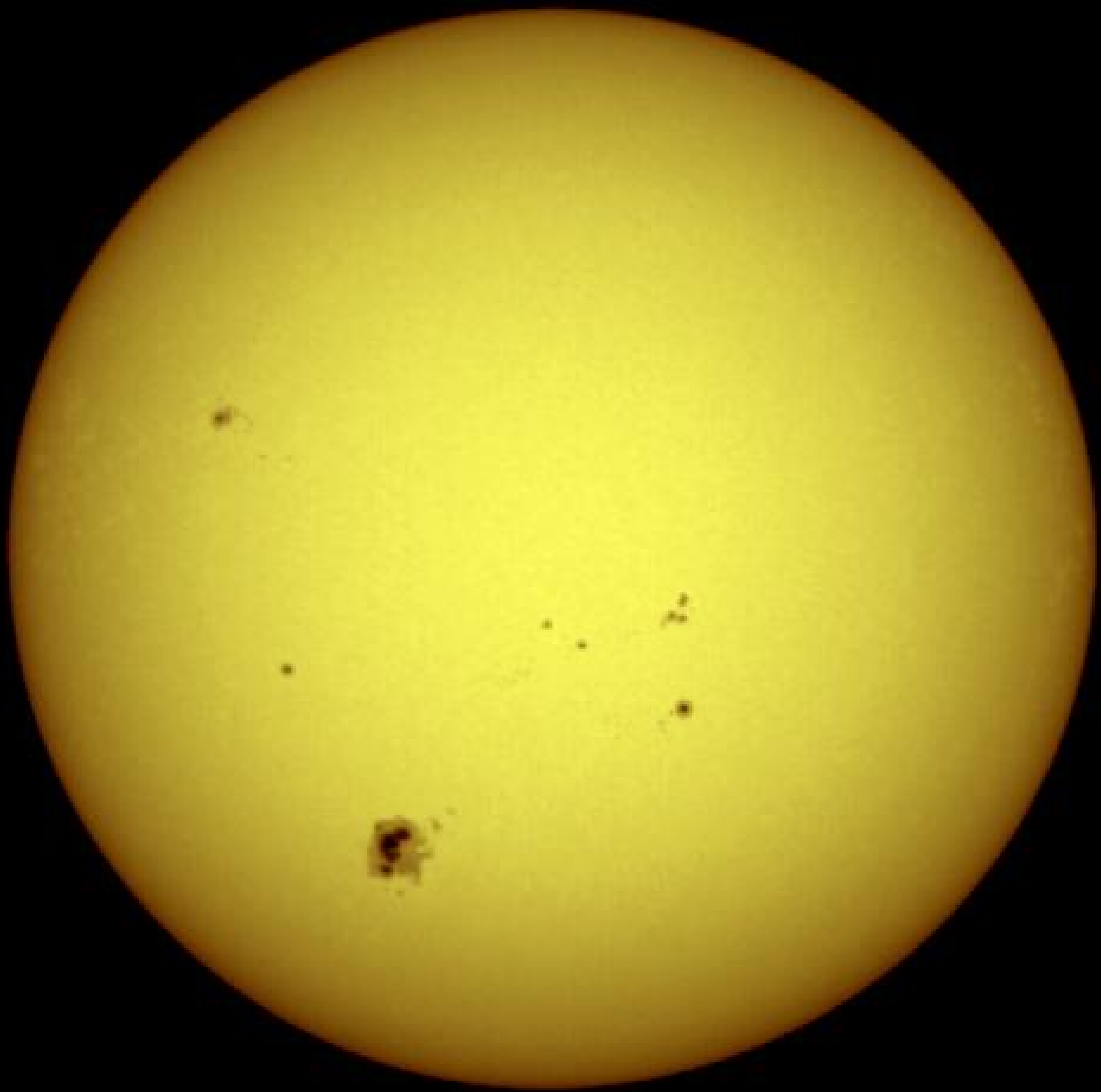


Image courtesy of NASA.

$M \sim 1/m_p^2$

Weisskopf 1975
Carr & Rees 1979

$$R \sim \alpha^3\beta / m_p{}^3\beta^{3/2}$$

$$M \sim \alpha^5 / m_p{}^3\beta^{1/2}$$

Carr & Rees 1979

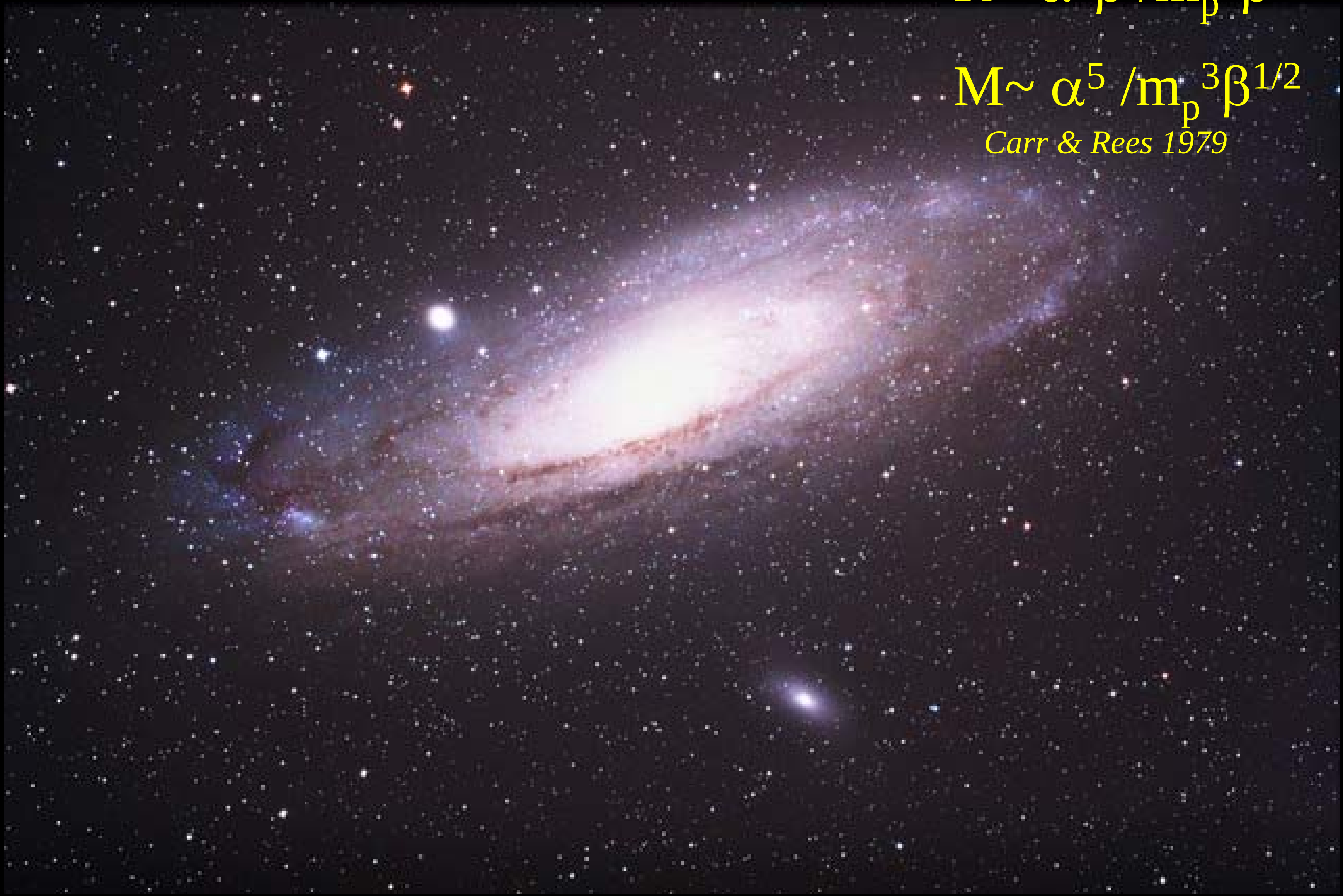


Image courtesy of Wikipedia.

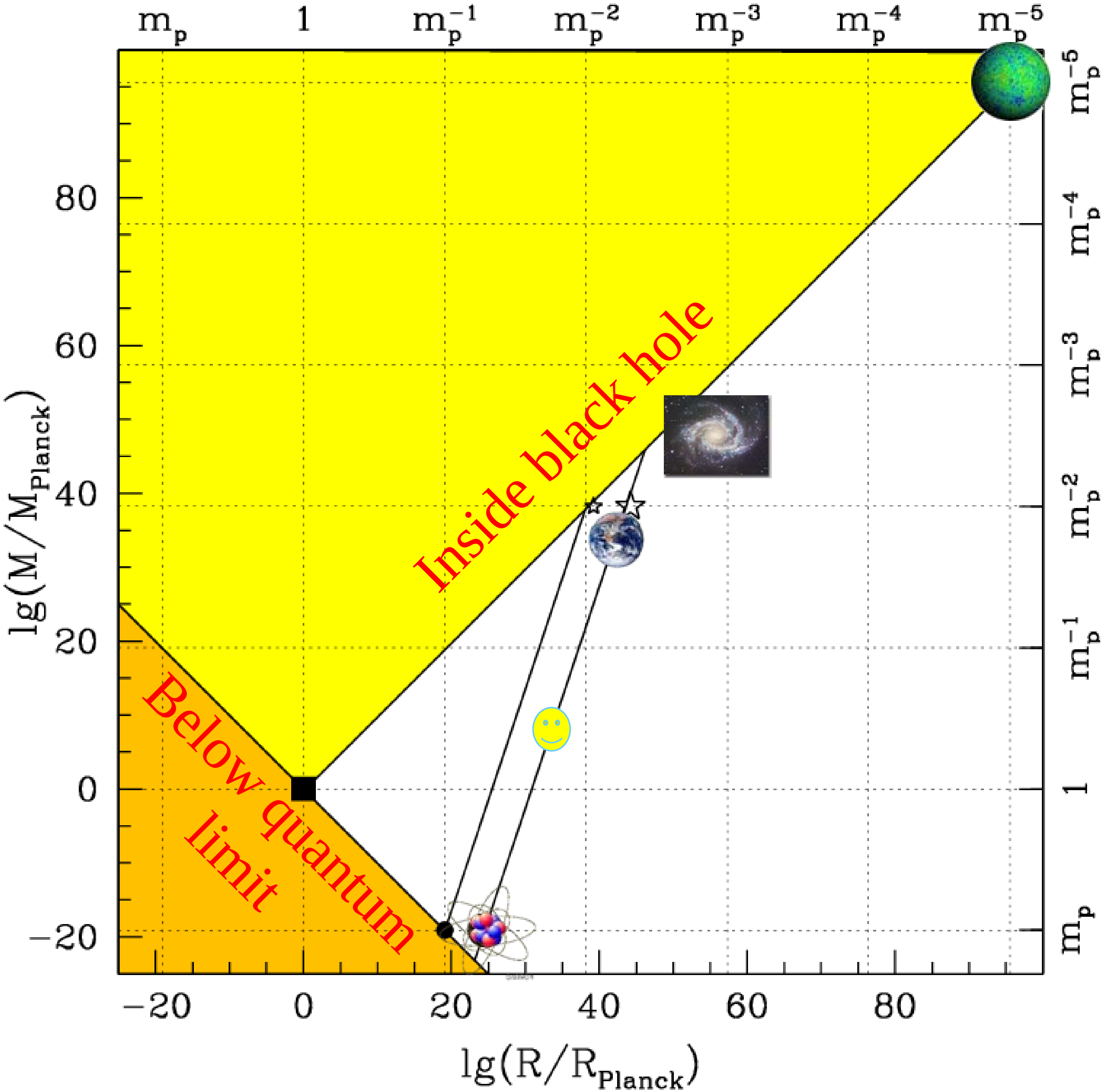
The known universe

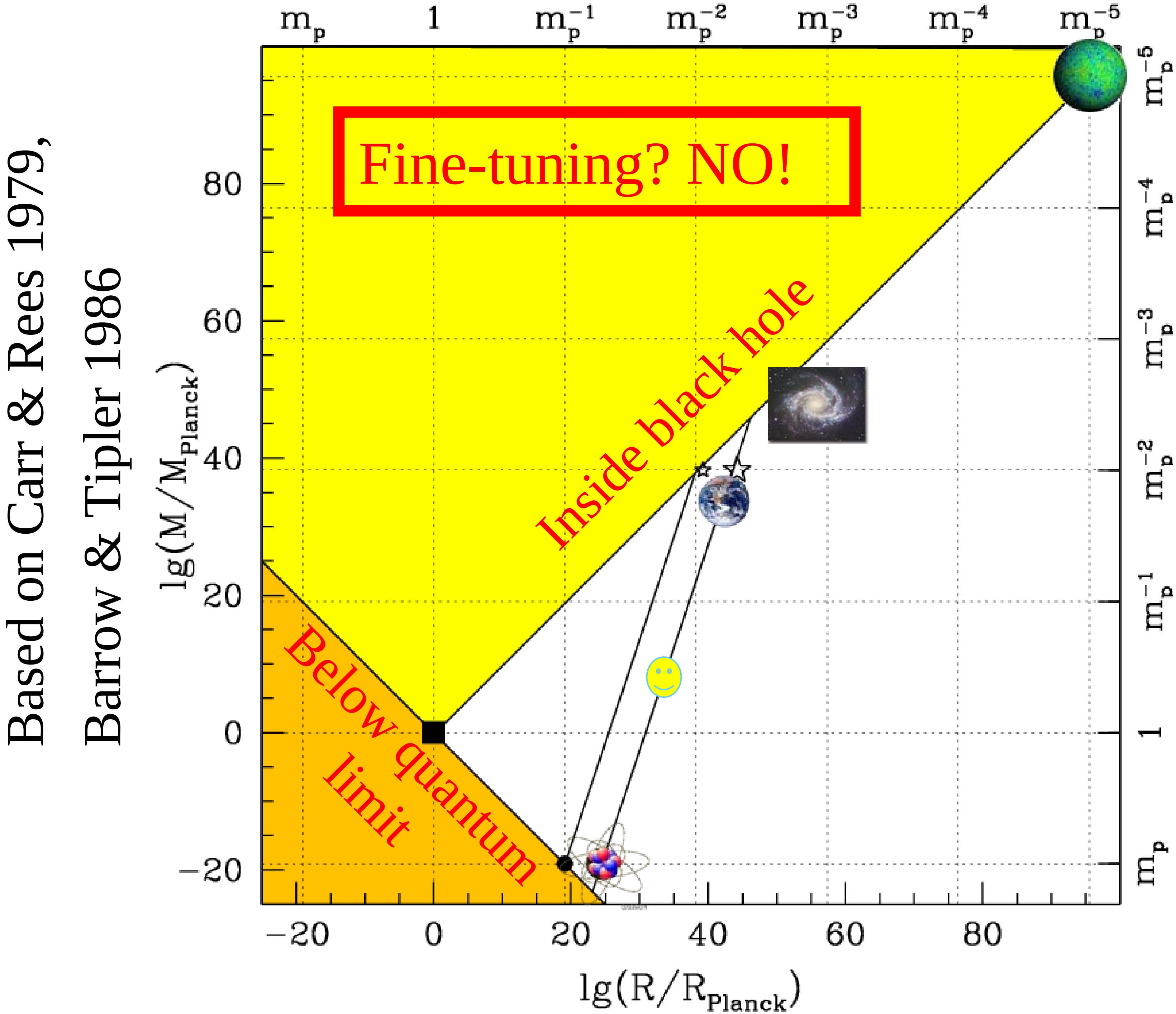
$R \sim 1/m_p^5$

$M \sim 1/m_p^5$

Carr & Rees 1979

Based on Carr & Rees 1979,
Barrow & Tipler 1986





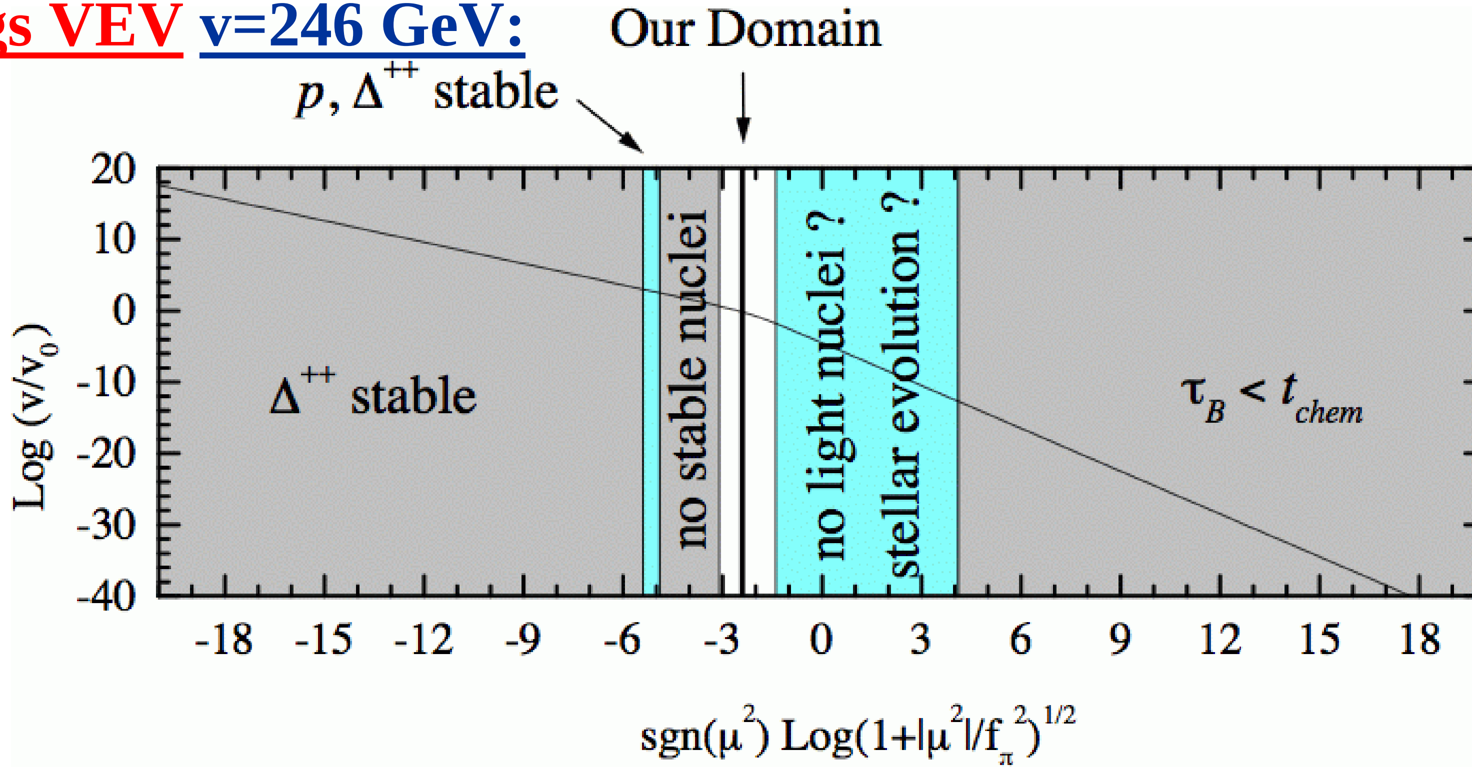
Based on Carr & Rees 1979,
Barrow & Tipler 1986
Fine-tuning? NO!
Inside black hole
Below quantum limit
lg(M/M_Planck)
lg(R/R_Planck)
m_p
1
m_p^-1
m_p^-2
m_p^-3
m_p^-4
m_p^-5
80
60
40
20
0
−20
80
60
40
20
0
−20

Figure 4 from Tegmark, “Is `the theory of everything' merely the ultimate ensemble theory?” http://arXiv.org/abs/gr-qc/9704009

Most spectacular fine tunings known:

- Dark energy density
- Higgs VEV

Effect of Higgs VEV v=246 GeV:



- $v/v_0<0.5$: protons (uud) decay into neutrons (udd)
- $v/v_0<0.8$: diproton & dineutron
- $v/v_0=1$: we are here
- $v/v_0>2$: deuterium unstable
- $v/v_0>5$: neutrons (udd) unstable even in nuclei
- $v/v_0>10^3$: protons (uud) decay to Δ^{++} (uuu)

Agrawal, Barr, Donoghue & Seckel 1998, *PRL*, **80**, 1822

$\alpha + \alpha \rightarrow {}^{8}Be$

${}^{8}Be + \alpha \rightarrow {}^{12}C$

${}^{12}C + \alpha \rightarrow {}^{16}O$

Figure 1 from Oberhummer, Csoto & Schlattl, “Stellar production rates of carbon and its abundance in the universe.” http://arXiv.org/abs/astro-ph/0007178

4 effective spatial dimensions:

no stable orbits, no stable atoms

(Ehrenfest 1917; Tangherlini 1963)

Figure 6 from Tegmark, “Is `the theory of everything' merely the ultimate ensemble theory?”
http://arXiv.org/abs/gr-qc/9702052

Figure 7 from Tegmark, “Is `the theory of everything' merely the ultimate ensemble theory?”
http://arXiv.org/abs/gr-qc/9702052