Physics 8.033   Out: Friday 22 September 2006
                                                Due: Friday 29 September 2006

Problem Set 3

Due: Friday 29 September 2006 at 4:00PM. Please Write your name deposit the problem set in the appropriate 8.033 bin, <u>**labeled by recitation section number and stapled**</u> as needed (3 points).

Readings: No new reading assignment in Resnick. Parallel reading: Chapter 5 in French.

Problem 1: (2004 Exam Question) (1+1+1+1 pts)

    Choose one answer for each of the following multiple choice problems. Circle your answer.

(a). A sphere is moving at 90% of the speed of light perpendicularly to your line of sight. In your rest frame, it is

    (a) a sphere.
    (b) a prolate ellipsoid (like a watermelon: One principal axis is longer than the other two.)
    (c) an oblate ellipsoid (like an M&M: One principal axis is shorter than the other two.)

(b). To you, it looks like

    (a) a sphere.
    (b) a prolate ellipsoid.
    (c) an oblate ellipsoid.

(c). A lamp is moving around you in a circle (you are at the center) at 50% of the speed of light. You observe it

    (a) redshifted.
    (b) blueshifted.
    (c) neither.

(d). An airplane is moving around you in a circle (you are at the center) at 50% of the speed of sound. You hear it

    (a) redshifted (lower frequency).
    (b) blueshifted (higher frequency).
    (c) neither.

Problem 2: "A Moving, Slanting Rod" (3+3 pts)

    A thin rod of length L', at rest in the S' frame, makes an angle of θ' with the x' axis, as in Figure 1.

    (a) What is its length L as measured by an observer in the S frame, for whom the rod is moving at a speed of βc in the direction of increasing x? (b) What angle θ does this moving rod make with the x axis? (b) Evaluate these quantities for $L' = 1.00$ m, $\theta' = 30°$, and $\beta = 0.40$.

Problem 3: "Decay in Flight" (3 pts)

    An unstable high-energy particle enters a detector and leaves a track 1.05 mm long before it decays. Its speed relative to the detector was 0.992c. What is its proper lifetime? That is, how long would it have lasted before decay had it been at rest with respect to the detector?
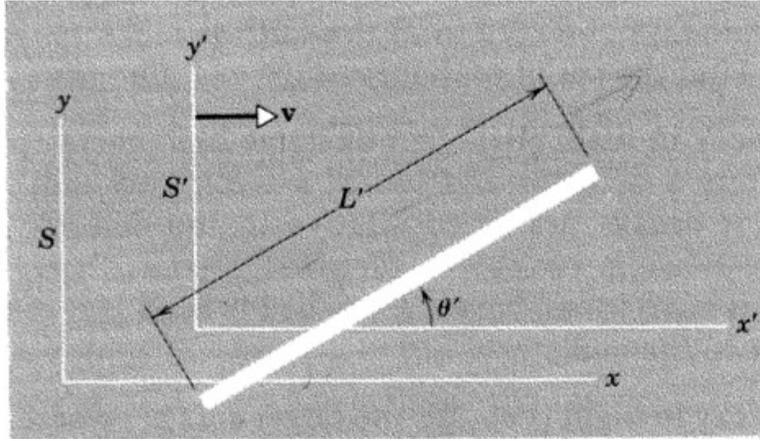
Figure 1:

Problem 4: "Pole Vaulter Problem": (3+3+3+3 pts)

A pole oriented in the x direction is at rest in the S' frame, which moves to the right with respect to the S frame at a speed $v = (3^{1/2}/2)c$, (i.e., $\gamma = 2$). The pole is 10 meters long in its own rest frame. It passes into a barn which is at rest in S with the right door closed and the left door open. The barn is 8m long in its own rest frame. At time $t = 0$, the left barn door closes with the left end of the rod ($x' = 0$) just inside the door. At that same time in S the right end of the rod ($x' = 10$m) is well inside the barn at $x = 5$m. Call the event ($x = 0, t = 0$) "A", and the event ($x = 5, t = 0$) "B". Thus, the barn doors are closed with the pole contained completely inside the barn. A short while later, the right door opens and the pole passes harmlessly out of the barn.

(a). Find t'_B in the vaulter's frame (assume that $t'_A = 0$).

(b). At t'_B in the vaulter's frame locate both ends of the rod in S.

(c). At $t'_A = 0$ in the vaulter's frame locate both ends of the rod in S.

(d). Sketch the rod and the barn in the S' frame at t'_A and t'_B, showing qualitatively how the pole vaulter understands the sequence of events.

Problem 5: "Lorentz-Einstein Transformation Along an Angle to the X-Axis" (6 pts):

A frame S' moves at constant speed v with respect to the S frame, but in a direction that makes an angle θ with respect to the x axis (see the diagram on the last page). Find the Lorentz-Einstein transformation between these two frames. Start with the Lorentz-Einstein transformations as we have derived them, i.e., with $\theta = 0$, and find the appropriate transformation when the relative motion of the two frames makes an arbitrary angle θ with respect to the x axis.

The easiest way to approach this problem is to carry out a rotation (rotate the velocity vector by an angle θ) in the S' frame until the direction of motion of the S' frame (relative to S) is along the new x' axis. Then the usual Lorentz-Einstein transformation can be applied to find x, y, and t. However, the S frame needs to be rotated by an angle $-\theta$ to get back to the original orientation shown in the sketch. Thus, the operations consist of a rotation, followed by a Lorentz-Einstein transformation, followed by a counter rotation. This can easily be carried out in linear algebra by using a product of three 3×3 transformation matrices. The rotation matrix is given by:

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5.1}$$

and the Lorentz-Einstein transformation matrix for the variables x, y, and t is:

$$\Lambda_x = \begin{pmatrix} \gamma & 0 & \gamma\beta \\ 0 & 1 & 0 \\ \gamma\beta & 0 & \gamma \end{pmatrix}. \tag{5.2}$$

To reverse the direction of rotation, simply change θ to $-\theta$. Therefore the operation described above is

$$\Lambda = R(-\theta)\Lambda_x R(\theta) \tag{5.3}$$

Problem 6: "Transforming Maxwells Wave Equation" (3+6 pts)

a) Using chain rule, derive how time and space derivatives $\frac{\partial f}{\partial t}$, $\frac{\partial f}{\partial x}$ of a function f transform under a Lorentz transformation to a reference frame moving with relative speed v along the x-axis.

b) Show that Maxwell's wave equation:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{6.1}$$

is invariant under a Lorentz transformation to a reference frame moving with relative speed v along the x-axis.

Problem 7: "Two Events - Same Time Difference, Same Separation, Different Sequence" (3+3 pts)

Observer S notes that two colored flashes of light, separated by 2400 m, occur along the positive branch of the x axis of his reference frame. A blue flash occurs first, followed after 5.00 μs by a red flash, the latter being the most distant from the origin of his reference frame. A second observer S' obtains exactly the same numerical values for the time diffrence and the absolute spatial separation between the two events but declares that the red flash occurs first. (a) What is the relative speed of S' with respect to S? (b) Which flash will S' find to be the more distant from the origin of her reference frame?

Problem 8: "An Event Pair - Timelike or Spacelike" (3+3+3+1 pts)

Two events occur on the x axis of reference frame S, their spacetime coordinates being:

Event	x	t
1	200 m	5.0 μs
2	1200 m	2.0 μs

(a) What is the square of the spacetime interval $(\Delta s)^2$ for these two events? (b) What is the proper distance interval $\Delta\sigma$ between them? (c) If two events possess a (mathematically real) proper distance interval, it should be possible to find a frame S' in which these events would be seen to occur simultaneously. Find this frame. (d) Can you calculate a (mathematically real) proper time interval $\Delta\tau$ for this pair of events? (e) Would you describe this pair of events as timelike? Spacelike? Lightlike?

Problem 9: "Transformation of Speeds": (3 pts)

A particle in the S' frame has velocity components u'_x and u'_y. In turn, S' is moving to the right with speed v. Find the speed of the particle in frame S. Express your answer as $u(u'_x, u'_y, v)$, without any explicit γ's being included. Show—in the limit of either u'_x, u'_y, or v approaching c—that u remains less than c. Also, see problem #62, chapter 2, page 87 (Resnick & Halliday) for an interesting form of the answer which makes the limiting speed u more obvious.

Problem 10: "Quasar, Quasar Burning Bright"(3+3 pts)

In the spectrum of the quasar 3C9, some of the familiar hydrogen lines appear but they are shifted so far forward toward the red that their wavelengths are observed to be three times as large as that observed in the light from hydrogen atoms at the rest in the laboratory. (a) Show that the classical Doppler equation gives a velocity of recession greater than c. (b) Assuming that the relative motion of 3C9 and the earth is entirely one of recession, find the recession speed predicted by the relativistic Doppler equation.

Problem 11: (2004 Exam Question)(3 pts)

The Starship Enterprise fires two identical laser guns in opposite directions. How fast and in what direction (draw a picture) is the Enterprise moving in your frame if in your frame, the two rays make an angle of 120 degrees and are observed to have the same frequency?

Problem 12: "The Ives-Stillwell Experiment" (9 pts):

Neutral hydrogen atoms are moving along the axis of an evacuated tube with a speed of 2.0×10^6 m/s. A spectrometer is arranged to receive light emitted by these atoms in the direction of their forward motion. This light, if emitted from resting hydrogen atoms, would have a measured (proper) wavelength of 486.133 nm. (*a*) Calculate the expected wavelength for light emitted from the forward-moving (approaching) atoms, using the exact relativistic formula. (*b*) By use of a mirror this same spectrometer can also measure the wavelength of light emitted by these moving atoms in the direction opposite to their motion. What wavelength is expected under this arrangement, in which the light source and the observer are -effectively- separating? (*c*) Calculate the difference between the average of the two wavelengths found in (*a*) and (*b*) and the unshifted (proper) wavelength. By this technique, Ives and Stillwell were able to distinguish between the predictions of the classical and the relativistic Doppler formulas.

Optional Problem 13: "To the Galactic Center!"

(*a*) Can a person, in principle, travel from earth to the galactic center (which is about 28,000 ly distant) in a normal lifetime? Explain, using either time-dilation or length contraction arguments. (*b*) What constant velocity would be needed to make the trip 30 y (proper time)?

Optional Problem 14: "What Time is it Anyway?":

Observers S and S' stand at the origins of their respective frames, which are moving relative to each other with a speed of 0.60c. Each has a standard clock, which, as usual, they set to zero when the two origins coincide. Observers S keeps the S' clock visually in sight. (*a*) What time will the S' clock record when the S clock records 5.00 μs? (*b*) What time will observer S *actually read* on the S' clock when his own clock reads 5.00 μs?

Optional Problem 15: "The Unreachable Goal":

A spaceship, at rest in a certain reference frame S, is given a speed increment of 0.50c. It is then given a further 0.50c increment in this new frame, and this process is continued until its speed with respect to its original frame S exceeds 0.999c. How many increments does it require?

Optional Problem 16: "A Philosophical Difficulty":

Suppose that A causes event B, the effect now being propagated from A to B with a speed greater than c. Show, using the relativistic velocity transformation equation, that there exists an inertial frame S', which moves relative to S with a velocity less than c, in which the order of these events would be reversed. Hence, if concepts of cause and effect are to be preserved, it is impossible to send signals with a speed greater than that of light.

Source: Problems 2, 3, 7, 8, 9, 10, 12, 13, 14, 15, 16 are taken from *Basic Concepts in Relativity* by Robert Resnick, David Halliday.
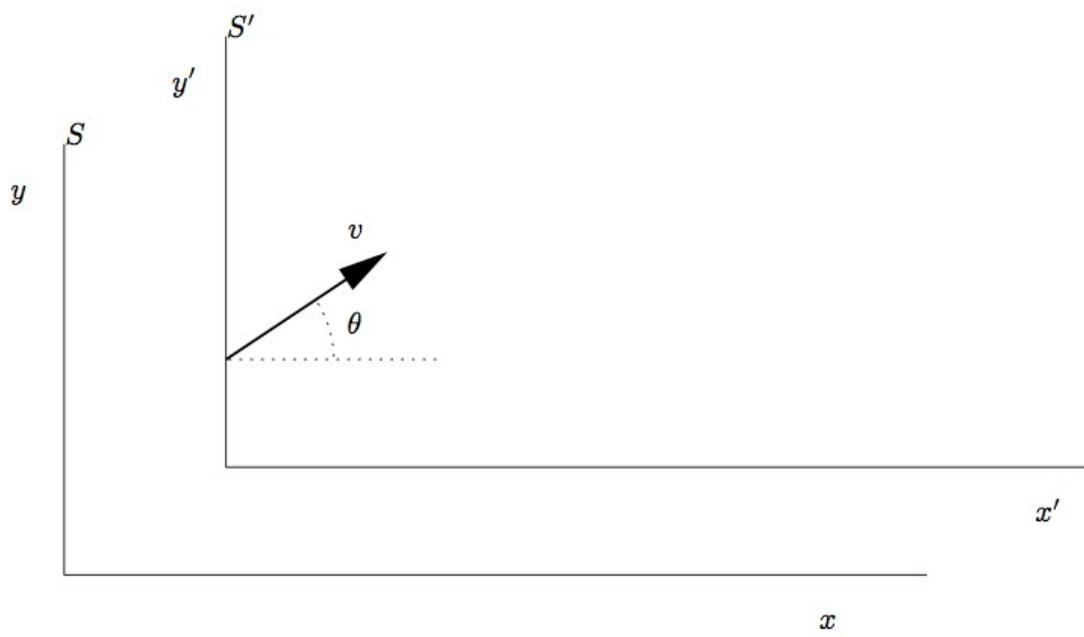
Feedback: Roughly how much time did you spend on this problem set?

Figure 2: Sketch for Problem 5.