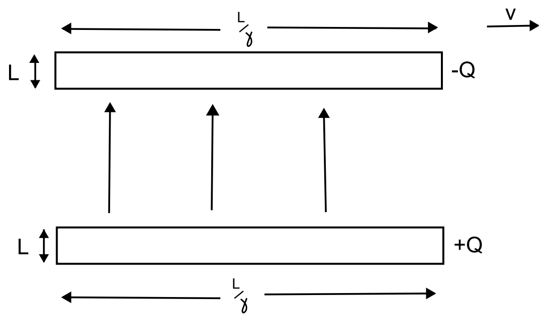
8.022 Lecture Notes Class 21 - 10/23/2006

$$x = \gamma(x' + vt') \quad \gamma > 1 \tag{1}$$
$$t = \gamma(t' + \tfrac{vx'}{c^2}) \tag{2}$$
$$y = y' \tag{3}$$
$$z = z' \tag{4}$$

Electric Fields

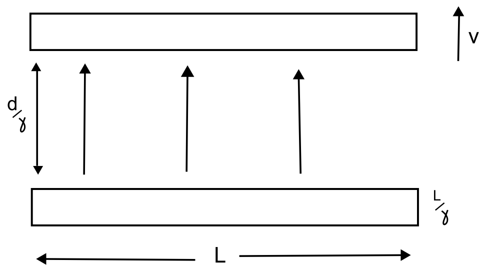


Accelerate the capacitor!

$$\sigma = \frac{Q}{L^2} \qquad \sigma' = \frac{Q\cdot\gamma}{L^2} = \gamma\sigma$$

$$\vec{E} \propto \sigma$$
$$\vec{E}' \propto \gamma\sigma$$

$$\vec{E_\perp}' = \gamma E_{perp} \quad \text{(perpendicular)}$$



$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} \text{ , so } \vec{E}_{||}{}' = \vec{E}_{||}$$

$$\begin{aligned} E' &= (E - vp_x) \\ p'_x &= \gamma(p_x - vE) \\ p'_y &= p_y \\ p'_z &= p_z \end{aligned}$$

<u>Force</u>

$$\begin{aligned} F_x &= \tfrac{dp_x}{dt} \\ \Delta x &= \tfrac{1}{2}a_x t^2 \\ &= \tfrac{1}{2}(\tfrac{F_x}{m})\Delta t^2 \end{aligned}$$

<u>Energy</u>

$$\begin{aligned} \Delta E &= \tfrac{(F_x \Delta t)^2}{2m} \\ &= \tfrac{1}{2}mv_x^2 \end{aligned}$$

<u>Force</u>

$$
\begin{aligned}
F'_x &= \frac{dp'_x}{dt'} \\
\Delta p'_x &= \gamma(\Delta p_x - v\Delta E) \\
&= \gamma(\Delta p_x - v\frac{(F_x \Delta t)^2}{2m}) \\
\text{From equation(2)} \qquad \Delta t' &= \gamma(\Delta t - \frac{v\Delta x}{c}) \\
&= \gamma(\Delta t)(1 - \frac{v\Delta t F_x}{2mc}) \\
F'_x &= \frac{\Delta p'_x}{\Delta t'} = \frac{\gamma(\Delta p_x - \frac{vF_x^2 \Delta t^2}{2m})}{\gamma \Delta t(1 - \frac{v\Delta t F_x}{2mc})} \\
&= \frac{\Delta p_x(1 - cF_x^2 \frac{1}{2m}\Delta t \cdot \frac{\Delta t}{p_x})}{\Delta t(1 - \frac{1}{2mc} v\Delta t F_x)} F'_x = F_x \cdot (1)
\end{aligned}
$$

$$\Longleftrightarrow F'_x = F_x \quad \text{(parallel)}$$

$$F'_y = \frac{\Delta p'_y}{\Delta t'} = \frac{\Delta p_y}{\gamma \Delta(1 - D\Delta t)} \qquad \text{D constant}$$

Take limit as $\Delta t \to 0$.

$$F'_y = F_{y/\gamma} \quad (parallel)$$

$$
\begin{cases}
a'_y = & a_y \\
m' = & m/\gamma
\end{cases}
$$

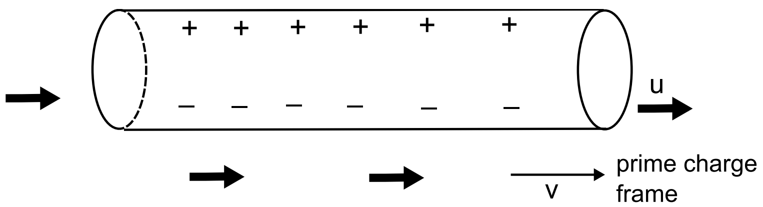
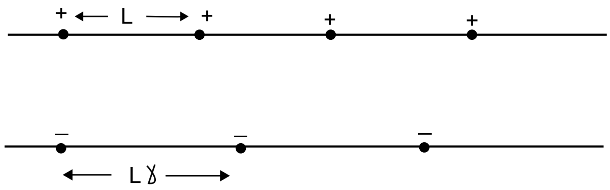


FIG. 1: Wire with moving negative charges

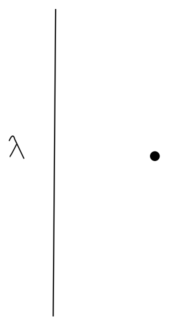
$$
\begin{aligned}
\lambda_+ &= \lambda_0 = \lambda_{+rest} \\
\lambda_- &= -\lambda_0 \neq \lambda_{-rest} \\
\lambda_{-rest} &= -\frac{\lambda_0}{\gamma}
\end{aligned}
$$

"Line" of moving negative charge



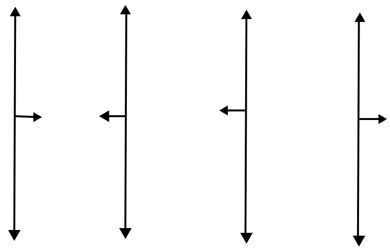
$v' = u - v$ (speed of neg. charge relative to moving prime charge)

$$
\begin{aligned}
\lambda'_+ &= \gamma_v \cdot \lambda_0 \\
\lambda'_- &= \gamma_v \cdot \lambda_{-rest} \\
&= \gamma_{v'} \cdot \frac{\lambda_0}{\gamma_u} \quad \gamma_{v'} = \gamma_u \gamma_v (1 - \beta_u \beta_v) \\
\lambda'_- &= -\frac{\gamma_{v'}}{\gamma_u} \lambda_0 \\
&= \gamma_u \gamma_v (1 - \beta_u \beta_v) \cdot \frac{\lambda_0}{\gamma_u} \\
&= \gamma_v (1 - \beta_u \beta_v) \lambda_0 \\
-\lambda'_- + \lambda'_+ &= -\gamma_v (1 - \beta_u \beta_v) \cdot \lambda_0 + \lambda_0 \cdot \gamma_v \\
&= \gamma_v \cdot \beta_u \beta_v \cdot \lambda_0 \\
&= \gamma_v \lambda_0 \frac{v \cdot u}{c^2} > 0 \quad \text{(Test charge flying , a charge on a wire)}
\end{aligned}
$$



$$
\begin{aligned}
E' &= \frac{\lambda}{2\pi\epsilon_0 r} \\
&= \frac{\gamma_v \lambda_0 \cdot v}{2\pi\epsilon_0 c^2} \cdot \frac{u}{r} \\
E' &= \frac{u\lambda}{2\pi\epsilon_0 c^2} \cdot \frac{\gamma_v v}{r}
\end{aligned}
$$

Assuming current in wire, there is electric field perpendicular to wire pulling on test charge
(reverse direction $\Rightarrow$ reverse electric field, since $-v$)



(u is opposite of current direction, remember)