

Topics: LC, and Undriven LRC Circuits

Related Reading:

Liao et al. Course Notes: Sections 11.5 – 11.6

Experiments: (7) Undriven & Driven LRC Circuits Part One: Undriven RLC Circuits

Topic Introduction

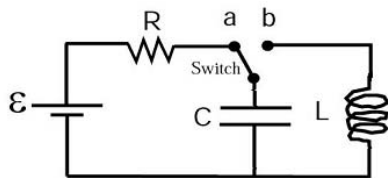
Today we investigate LRC circuits. We will see that the current in these circuits oscillates, in a fashion completely analogous to the oscillation of a mass on a spring

Mass on a Spring: Simple Harmonic Motion

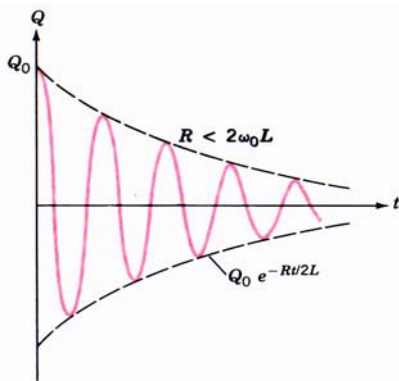
In a simple system consisting of a mass hanging on a spring, when the mass is pulled down and released it oscillates up and down. We think about this in a couple of ways. One way is to look at the forces on the mass and write a differential equation for its motion, $F = m\ddot{x} = -kx$, where \ddot{x} means two time derivatives of the displacement (acceleration). The solution to this is simple harmonic motion: $x = x_0 \cos(\omega t)$ where $\omega = \sqrt{k/m}$.

We can also think about the energy in the system. As the mass moves, energy oscillates between kinetic energy of the mass and potential energy stored in the spring. If there is no damping (friction) in the system to dissipate energy, the oscillation will continue forever.

Undriven L(R)C Circuits



Consider the LC circuit at left, where the switch is at “a” until the capacitor is fully charged and then thrown to “b.” This is analogous to pulling down a mass and releasing it. Here the capacitor will want to discharge and will drive a current through the inductor. Eventually all the charges will run off of the capacitor (spring), so it won’t “push” anymore, but now the inductor will want to keep the current flowing through it that it already has (inductors, like masses, have inertia). It will keep the current flowing, but that will eventually fill up the capacitor which will stop the current and send it back the other direction. Our differential equation is thus analogous, $V = -L\ddot{q} = q/C$, and has the same solution: $q = q_0 \cos(\omega t)$ where $\omega = \sqrt{1/LC}$.



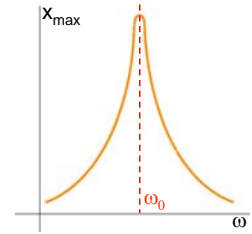
We can also think about energy here, where it oscillates between being stored in the electric field in the capacitor and the magnetic field in the inductor. As long as there is no dissipation (resistance) in the circuit the oscillations will continue forever.

If we add a resistor in series with the capacitor and inductor we provide a method of energy loss, through joule heating in the resistor as current flows. The oscillations will thus damp out to zero. The exact path the charge will take as it

oscillates to zero depends on the relative sizes of L, R and C, but will typically look something like the curve above, where the oscillations are bounded by an “envelope” which is exponentially decaying to zero as a function of time.

Driven LRC Circuits

We can also add a force to our circuits – the AC power supply. In this case the current responds at the drive frequency. However, depending on the frequency of the drive, the current may be out of phase (either *leading* or *lagging* the drive) and its amplitude will also vary. This is easily seen in mechanical systems. For a fantastic example, play with the pendula at the Kendall T station. Depending on how fast you drive them they will respond either in or out of phase with your drive, and they will either move a little or a lot. When you drive at the natural frequency, the amplitude increases greatly, and the system is “in resonance.”



One Element at a Time

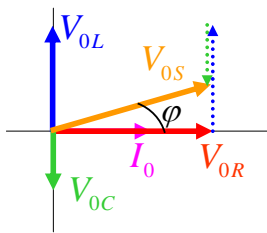
In order driven LRC circuits its easiest to start thinking about individual circuit elements. A resistor obeys Ohm’s law: $V=IR$. Neither the amplitude nor phase of the voltage depends on the frequency (the response voltage is always in phase with the current).

A capacitor is different. At low frequencies the capacitor “fills up,” but at high frequencies it begins discharging before “filling up.” The voltage is frequency dependent and the current *leads* the voltage (current flows before the charge/potential on the capacitor appears).

An inductor is the opposite – it hates the change of high frequencies and thus has large voltages there – and the current *lags* – the inductor fights before current flows.

When put together in LRC circuits, the capacitor “dominates” at low frequencies, the inductor at high ones. At resonance ($\omega = \sqrt{1/LC}$) the frequency is such that these two effects balance and the current will be largest in the circuit. Also at this frequency the current is in phase with the driving voltage (the AC power supply).

Seeing it Mathematically – Phasors



To do this mathematically we will use phasor diagrams. A phasor is a vector whose magnitude is the amplitude of either voltage or current and whose angle corresponds to its phase. Phasors rotate CCW about the origin with time as their phase evolves, and their current amplitude is their component along the y-axis, which oscillates as it should.

Phasors allow us to add voltages that are not in phase with each other.

For example, the phasor diagram above illustrates the relationship of voltages in a series LRC circuit. The current I is assigned to be at “0 phase” (along the x-axis). The phase of the voltage across the resistor is the same. The voltage across the inductor L leads (is ahead of I) and the voltage across the capacitor C lags (is behind I). If you add up (using vector arithmetic) the voltages across R, L & C (the red and dashed blue & green lines respectively) you arrive at the voltage across the power supply. This then gives you a rapid way of understanding the phase between the drive (the power supply) and the response (the current) – here labeled ϕ .

Important Equations

Natural Frequency of LC Circuit: $\omega_0 = \frac{1}{\sqrt{LC}}$

Experiment 7: Undriven and Driven LRC Circuits: Part One Undriven RLC Circuits

Preparation: Read pre-lab and answer pre-lab questions.

This lab consists of two main parts. In the first part (today) you will study an undriven LRC circuit and determine its natural frequency. You will do the second part in the next class.