We would now like to calculate the potential function for a universal law of gravity.

So let's set up a coordinate system first.

And in this case, we're going to make a very special case-- although, the general result doesn't depend on that.

So suppose we have a planet of radius rp and we have a small object.

And initially, our object is a distance r away from the planet.

This is our initial state.

And the planet has mass to the planet.

And our small object has m.

And this object is moving.

Of course, orbits really aren't like that, so we should be a little bit more careful.

But our orbit might be some type of hyperbolic section-- conic section.

And the orbit over here is at some final position.

In our eyes, the distance from the object to the center of the planet as our final is.

Now because there's a central point to this problem, the natural coordinate system to use is polar coordinates in the plane.

And let's just imagine that at this moment we have the universal force of gravity.

Here's our object.

And it's being displaced a distance s.

So our coordinate system is polar.

Let's call r hat this way and theta hat that way.

And let's blow up our little displacement in terms of our coordinate system, just so that we see what's happening.

So let's imagine that our little ds vector is pointing like this.

And our center point here, we have tge r hat pointing radially outward and the theta hat pointing towards the center.

Now to exaggerate this picture, if this is r, then the arc length here is rd theta.

And the difference as the object moves from one point here to a closer point towards the center, will be dr.

And so our ds vector, we can write as a radial piece r hat.

Now, I don't put a sign in there, because only the end points of my integral will tell me whether I was going towards the planet or away from the planet.

And rd theta, theta hat.

And our gravitational force is minus g mass mass of the planet over the distance squared r hat.

So now what we see is that when we take the dot product of these two forces, r hat dot r hat is 1 in polar coordinates.

Why is that the case, r hat dot r hat is 1?

Because these vectors are in the same direction, the angle between them is 0.

And remember, that any two vectors that are perpendicular have dot product 0.

r hat and theta hat are perpendicular.

And so r hat dot r hat is 1.

r hat dot theta hat is 0.

And so we only get minus GMP r squared dr.

And that's the first step in calculating our potential difference because U of r final minus U of r initial by definition is minus the work done in going from the initial state to the final state, which are described by these parameters r initial and r final of F gravitation dot ds.

And now we have, actually, minus sign in the definition, another minus sign coming from the dot product.

And lets make this our integration variable, r prime, r initial, r prime equals r final.

And there's going to be a third minus sign because the integral of dr r prime squared is minus 1 over r prime.

So there would be 3 minus signs-- one from the definition, one from the scalar product, and one from the integration.

And so we get r final minus r initial equals 3 minus signs given overall minus sign planet and planet 1 over r final minus 1 over r initial.

So this is the change in gravitational potential energy as my small object goes from some initial state to some final state.

What about the potential function?

Where should we choose our reference point?

So our reference state here is a little bit unusual.

And it will be at infinity.

And we'll choose as a potential for our reference potential to be 0 at infinity.

Imagine initial state very, very far away from the planet.

And we'll take us a final state, an arbitrary state, just where the object is some distance r from the planet.

Now, we have to be a little bit careful, because in this calculation our final state must always be outside the planet.

And the reasons for that is subtle, but the gravitational force inside the planet is no longer minus g and 1 M2 over r-squared.

So our analysis only applies for r bigger than r planet.

And if we put these values for r and infinity in here, we get U of r minus the potential at our reference point, which will be 0, equals minus GMNP.

And here's the reason why we choose that reference point, 1 over r arbitrary state is just minus 1 over r.

But 1 over infinity, if we use the initial state as the reference point, 1 over infinity is 0.

So that's minus 0.

And so what we get is that the potential function is equal to our reference potential-- actually, minus GM mass of the planet over r for r bigger than our reference.

And that is the potential function for the gravitational problem.

But now remember, we're choosing that to be our reference point.

And so the real conclusion is U of r equals minus GMN planet over r with U at infinity equals to 0.

Now, let's just finish this by saying in words precisely what this means.

So if I start my system at infinity and I end my system a distance r away from the planet, and I calculate the work done by the gravitational force in going from infinity to a distance r away from the planet-- I take a minus that sign.

So the negative of the gravitational work-- then that's what that number corresponds to.