We already defined work in one dimension is the product of force times displacement for a constant force, but now let's look at a case where we're applying a force, f, that is a function of x.

So, our component, f of x, is a function of x in the i hat direction.

And one of the simplest examples of a force like this is what we call the spring force.

Now, when we apply this force to our object-- and let's look at an object, i, i hat direction, we'll have an origin.

And this is our plus-x coordinate system.

When we apply force-- what we want to look at is because the force is a function of position-- then we want to look at the displacement over a small amount.

So let's call this the point xj.

And out here, let's refer to this as xj plus 1.

And what we've done is we're going to ask how much work is done when the force is displaced from here to there.

And that's what we'll call delta x.

So now our displacement is a small displacement x of j plus 1 minus x of j.

And our work for this small displacement-- and that's why we'll indicate it with a delta-- is equal to the force, which is a function of x, times this displacement.

And so we get f of x times x of j plus 1 minus x of j.

And what we have to indicate here is because the force is varying, we're looking at just this displacement here.

Let's refer to this force as in the j-th part.

The total work is just the sum of all these scalar quantities.

Remember, although force is a vector and displacement is a vector, the product of these two quantities is a scalar.

And so, if we want the total work, we have to sum from j goes from 1 to n of this quantity f of j dot-- and I'll put a little j there to indicate that-- delta xj.

What does this sum mean?

Well imagine that we're making a series of displacements all the way out to a final position, x final, and we divided this interval into n pieces.

And so this represents the little bit of work done for all of these displacements.

And that is what we define to be the work for a non-constant force.

The issue here is about how fine we cut this interval in.

We made n individual pieces.

But if we want to ask ourselves, what is the limit as n goes to infinity, then that's what we now need to consider.

So what we're doing is we're making smaller and smaller and smaller little displacements.

And we're taking this sum-- j goes from 1 to n of x of xj times delta xj.

And this limit of a sum is by definition the integral of the force with respect to dx.

So what we end up with is our work is the integral of f of x.

It's a function and I'm going to have an integration variable, x prime of dx prime, where x prime is going from our initial position to our final position.

And this is now our definition of work which generalizes a constant force to a non-constant force.

Again, let's try to look at some type of geometric interpretation.

So if we plotted f of x versus x.

And now let's consider a case where we have some arbitrary force.

So I'm going to just draw the force as if it were arbitrarily increasing as a function of x.

And here is our x initial.

And here's our x final.

And again, we would like to make a geometric interpretation.

Let's consider xj.

And I'll make this very big.

xj plus 1 for the sake of visualization.

And this value here is f of xj.

And so what we see now is that little bit of work, like we had for a constant force, can be approximated as the area underneath the curve for just this small interval between xj and xj plus 1.

And if we make this sum, what we're doing is we're just taking a series of approximations to the area under these curves.

Now you can see, graphically, that there is very little error when the function was nearly constant.

But in this position, where the function is growing, this represents our little error.

However, in the limit, as n goes to infinity, we can make that error go vanishingly small.

So once again, we see, as a geometric interpretation, that this is the area under the curve of the force versus position between the points initial xi and initial-- and the final, x final, where the particle is being displaced from an initial position to some final position.

And here we can call our initial position anywhere we want.

If I write that, this x initial, to the final position.

And that's our generalization of the definition of work.