

We'd like to consider a system of particles.

Let's say we have object one.

We'll call this the  $j$ -th object, object  $n$ .

And somewhere in this system of particles is a center of mass.

Now, we know that the external force is causing the momentum of the system to change, and what we'd now like to show is that we can reduce this system to an effective single particle.

The way we'll do that is recall that the momentum of the system is given by the sum of the individual momentums.

We'll call that the  $j$ -th particle where we're summing  $j$  from 1 to  $n$ .

And that's the sum of the momentums of the various particles.

Now when you differentiate the momentum of the system with respect to time, we're just differentiating the velocity.

And so that becomes the acceleration of the  $j$ -th particle.

Now we saw when we define the center of mass that the acceleration of the center of mass times the total mass of the system was equal to the sum of  $m_j a_j$ .

$j$  goes from 1 to  $n$ .

So we see that another way to think about how the momentum of the system of particles is changing is simply the total mass times the acceleration of the center of mass.

And our combination of Newton's second and third law now becomes that only the external forces cause the momentum of the system to change so that's equal to the total mass times the acceleration of the center of mass.

Now what does this equation really mean?

So let's draw our pictures again.

Here's our system.

We have particles 1, et cetera.

That's the  $j$ -th particle,  $n$  particles.

And in here is the center of mass.

Now I'm going to outline my system like this and what this equation is telling us is that we can just focus by putting all of the mass and total at the center of mass.

And that center of mass is going to move according to some trajectory.

And all we have to think about is this is a point particle of total mass  $m$ ,  $m$  total.

So what we've done is we've done a very important reduction.

We've taken a complicated system of particles and reduced it to a single point particle of total mass  $m$  located at  $\mathbf{cm}$ .

And the dynamics of that total particle is if there is external force acting on this system, we place this external force at the center of mass.

And now we can calculate the acceleration of the center of mass is just that external force divided by the total mass.

And that's how we can reduce this complicated system of particles to a translational motion of the center of mass.

Now we still cannot describe the individual interactions in the system, but we're not trying to do that anymore.

We're not trying to trace how each particle moves.

We're just looking at as our system is a point particle and talking about how that point-like object is translating the space.

And this is a powerful tool that we use again and again and again.