I would now like to show you how to calculate the moment of inertia of a typical continuous body.

Let's consider a rigid rod, very thin.

And what we want to do is calculate the moment of inertia of this body about the center of mass.

Let's say the body is of length L, and it has total mass M.

Now, recall that the moment of inertia about the center of mass we defined as an integral of dm r-squared integrated over the body.

Now, our challenge today is to understand exactly what all these terms are in this expression.

What is dm?

What is r?

And what do we mean by an integral over the body?

So now let's do a stepwise interpretation of each term.

The crucial thing is to introduce coordinates systems.

So let's choose a coordinate system.

And let's put the origin at the center of mass.

Now, the most important thing is that we're going to do an integral.

So we need to introduce the integration variable.

And that's the hardest part of setting up intervals.

So what we want to do is arbitrarily choose an element dm.

So there's our elements dm.

And here's our integration variable, it's a distance x from the origin.

So that's the integration variable.

Now, there's two things when we set up the integral.

r is equal to that integration variable.

r is abstract in this expression, but in this concrete realization it is the integration variable.

The second place the integration variables shows up is in dm.

dm is a mass in this small element.

But if we want to express that in terms of our integration variable, we have to express it in terms of the differential length dx.

So dm mass is equal to the total mass per unit length.

We're assuming the rod is uniform times the length dx of our small piece.

And now we've set up the two pieces that are crucial and all we have to think about now is what does an integral mean.

Well, an integral means that we're dividing up the piece into a bunch of small elements and we're adding the contribution of each small element.

So in particular, when we write Icm equals-- now, we can write it as m over Ldx.

That was our dm.

And the distance of dm from the point we're computing the axis is x-squared.

Now, the question is what is our integration variable doing?

Well, x is going from minus L over 2-- that's at this end-- to x equals plus L over 2 on the other end.

So we have x minus L over 2x equals plus L over 2.

And now we've set up the integral for the moment of inertia, and the rest is just doing an integral.

Recall that the integral of dx x-squared is x-cubed over 3.

And so this integral then simply becomes Icm equals m over L x-cubed over 3-- evaluating from the limits minus L over 2 to x equals plus L over 2.

Again when you evaluate the limits, what we get is m over L we have to put in L over 2 cubed divided by 3 minus L over 2 cubed divided by 3, and that's 1 over 2 cubed is an eighth-- divide by third, that's the 24.

24 minus 24 is a 12.

And so what we get for Icm is m over L, 12 L-cubed, or 1/12 m L-squared is the moment of inertia about the center of mass of our rigid rod.

And this is a measure of how the mass is distributed about this axis.