- Last Lecture
  - Power, Impulse, Center of Mass
- Today
  - Simple Harmonic Motion
- Important Concepts
  - The physics of the motion is in the mass and spring constant which determine the period of each oscillation.
  - The amplitude does not affect the period.
  - Energy oscillates between Kinetic and Potential.

Important Reminders

- Pset # 8 due tomorrow.
- Problem Solving session in class tomorrow.
- MasteringPhysics due next Monday.
  - NOTE: Class grading guidelines clearly allow discussion of MasterPhysics problems but also clearly prohibit directly working together or copying the answers of others.
- No Pset due next week.
- No formal tutoring sessions next week.

Center of Mass Velocity

- Definition: $\vec{v}_{C.M.} = \frac{1}{M_{TOT}} \sum m_i \vec{v}_i$
- Connection to momentum: $M_{TOT}\vec{v}_{C.M.} = \vec{p}_{TOT}$
- So, if momentum is conserved, the velocity of the center of mass is constant.

Simple Harmonic Motion - I

- Start with Force equation: $\vec{F}_{Spring} = -k(\vec{l} - \vec{l}_0)$
- Define x axis along direction spring is stretched and put x=0 at the point the spring is unstretched:

$$F_x = -kx = ma_x = m\left(d^2x/dt^2\right)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- So, what is the solution to the differential equation?

Simple Harmonic Motion - II

- The answer is sine and/or cosine function with three mathematically equivalent ways to write it:

$$x = A\cos(\omega t + \phi)$$
$$x = A\sin(\omega t + \phi)$$
$$x = A\cos(\omega t) + B\sin(\omega t)$$

- In all cases, A, B, and ϕ are constants determined by the initial conditions.
- ω is given by the physics: $\omega = \sqrt{\frac{k}{m}}$

Simple Harmonic Motion - III

- Connections to the physical motion:
  - A is the amplitude, the maximum displacement from zero
  - ϕ is an arbitrary constant that depends only on when you define t=0, no real connection to the nature of the motion
  - ω is angular frequency in radians per second:

$$\text{Frequency in cycles/second} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\text{Period (time for one cycle)} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Velocity/Acceleration in SHM

- Also sine/cosine functions:

$$x = A\cos(\omega t + \phi)$$
$$v_x = {dx}/{dt} = -A\omega\sin(\omega t + \phi)$$
$$a_x = {dv_x}/{dt} = -A\omega^2\cos(\omega t + \phi)$$
$$= -\omega^2 x = -\frac{k}{m}x$$

- Note that: Maximum speed $|v_{Max}| = A\omega$

Energy in SHM

- Spring PE: $PE_{spring} = \frac{1}{2}kx^2 = \frac{kA^2}{2}\left(\cos(\omega t + \phi)\right)^2$
- Kinetic energy: $KE = \frac{1}{2}mv^2 = \frac{m(A\omega)^2}{2}\left(\sin(\omega t + \phi)\right)^2$
- Total:

$$E_{Total} = KE + PE = \frac{kA^2}{2}\left(\cos(\omega t + \phi)\right)^2 + \frac{m(A\omega)^2}{2}\left(\sin(\omega t + \phi)\right)^2$$
$$\omega^2 = {k}/{m}$$
$$E_{Total} = \frac{kA^2}{2}\left(\left(\cos(\omega t + \phi)\right)^2 + \left(\sin(\omega t + \phi)\right)^2\right) = \frac{kA^2}{2} = \frac{1}{2}mv_{Max}^2$$