22.01 Fall 2015, Problem Set 7 (Normal Version)

Due: November 20, 11:59PM on Stellar

December 11, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1   Conceptual/Analytical Questions

1. Explain, using your knowledge of criticality (the four and six factor formulas) and nuclear reactions, why water makes a fantastic moderator.
   Water is an excellent moderator, due to a combination of its nuclear (cross section) and thermal-hydraulic properties. Hydrogen $\left(^1H\right)$ has quite a low cross section, while oxygen $\left(^{16}O\right)$ has the lowest capture cross section of any element. In addition, the high density of hydrogen in water means that most collisions that happen have the potential to immediately slow down neutrons in an elastic collision, making them skip over the high-absorbance resonance energy regions. It is also generally non-corrosive, has a huge heat of vaporization (good for heat transfer), and has one of the highest heat capacities of any fluid. Its low viscosity also makes it great for pumping around a reactor with low frictional losses.

   (a) What could you do to the reactor to increase moderation without changing the coolant?
      There are a few ways: (1) Have more of it there (increase moderator to everything else ratio), (2) Make the water denser by cooling it down, (3) suppress boiling (which would lower the moderator density) by increasing the system pressure, like in a pressurized water reactor (PWR).

   (b) Can you think of a material that would moderate neutrons better than water, and why?
      A material's moderating power is a function of how much is there, and its value of α, or how much it can slow neutrons down. Therefore, finding a way to increase the hydrogen density, like by using liquid hydrogen (not very feasible) or using a solid metal-hydride (much more feasible) could pack more hydrogen into a tighter space.

   (c) For what reasons do some reactors (such as CANDU reactors) use heavy water as a moderator instead of light water? What disadvantages does it have?
      Looking at the absorption (n, γ) cross sections of H_2O vs. D_2O, one can see that while normal water may be a more effective moderator per se, normal water has a thermal absorption cross section of 0.11 b, while heavy water has a thermal cross section of 0.05 b. This means that heavy water absorbs fewer neutrons, which allows the enrichment of the fuel in the CANDU reactors to be low, even being able to use natural uranium! Its main disadvantages are (1) it creates tritium, a very radioactive isotope of hydrogen, and (2) it is very expensive compared to normal water.

2. Consider the fission of ^{235}U in these questions:

(a) Analyze the likelihood of forming each of these fission products, by computing the Q-values of the nuclear reactions that form each of these fission products by a thermal neutron: ^{8}Be, ^{16}O, ^{32}S, ^{64}Zn, ^{128}Te. How do these compare to the fission product yields in figure 12.12?

The following reactions were found using the KAERI table of nuclides, assuming that each takes the following form:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow 2^{1}_{0}n + FP_1 + FP_2 + \gamma \tag{1}$$
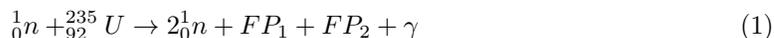
First Product	Second Product	Q (MeV)
^{8}Be	^{226}Ra	**4.2**
^{16}O	^{220}Rn	**29.2**
^{32}S	^{208}Os	**N/A**
^{64}Zn	^{170}Sm	**N/A**
^{128}Te	^{106}Zr	**181.5**

None of the reactions shown here are likely, except for the tellurium-based one. The first two have positive Q-values, but are extremely unlikely because the unequal nuclei of the fission products would have to overcome the strong-force barrier keeping them together. The last reaction is quite likely, as the two fission products are located near the maxima of Figure 12.12, the fission product distribution graph for U-235.

(b) For the case where one of the fission products is ^{128}Te, what is the average kinetic energy of the neutrons emitted following fission? You may assume that all the daughter products, including the neutrons, are emitted at equal angles from each other. For example, if two neutrons are emitted, the four daughters leave the compound nucleus at 90 degrees.

For this problem we will assume that the fission products and neutrons all leave at 90 degrees to each other. We will also assume that each carries away an equal amount of momentum, therefore it actually doesn't matter which particles exit in which direction! In both cases:

$$p_n = p_{Zr} \approx p_{Te} \tag{2}$$

We also know that the total kinetic energy of all particles must equal the Q-value, ignoring gammas, neutrinos, and anything else:

$$Q = T_{n_1} + T_{n_2} + T_{Zr} + T_{Te} \tag{3}$$

Let us also assume that each set of one neutron and one fission product has about half the kinetic energy of the whole equation:

$$\frac{Q}{2} = T_{n_1} + T_{Zr} \tag{4}$$

Using Equation 2, we can construct an equation relating the kinetic energies of the two particles:

$$p^2 = 2mT; \quad \cancel{2}m_n T_n = \cancel{2}m_{Zr}T_{Zr}; \quad T_{Zr} = \frac{m_n T_n}{m_{Zr}} \tag{5}$$

Substitute this into Equation 4:

$$\frac{Q}{2} = T_n + \frac{m_n T_n}{m_{Zr}}; \quad \frac{Q}{2} = T_n\left(1 + \frac{m_n}{m_{Zr}}\right); \quad T_n = \frac{Q}{2}\left(\frac{m_{Zr}}{m_n + m_{Zr}}\right) \approx 89.9\,MeV \tag{6}$$

This means that simple conservation of energy and momentum does NOT explain the neutron's true energy of 1-10 MeV. Thinking back to the sequence of events occurring in fission, first two very asymmetric fission products are made, which then quickly *decay* by neutron emission.

3. For these questions, consider the ability to induce fission in ^{84}Kr, and look through the ENDF/B-VII.1 cross sections in the JANIS cross section database:

   (a) Calculate the incoming neutron energy needed to induce the reaction of ^{84}Kr that releases one neutron at the least excited state (n,n'1, MT=51). Compare this with the energy at which the cross section for fission releasing one neutron begins from the JANIS database.
   The incoming neutron energy needed to induce the reaction of Kr-84 needed to release one neutron from the excited state is just the lowest-possible Q-value for inelastic scattering. Looking up the excited states[1] of Kr-84, we see that the first excited state is 881.6 keV. Looking up the first non-zero cross section energy in the Janis database, we see an energy of 892.5 keV. This could be due to energy level splitting, as the two neutrons at this energy would have different spins, and therefore slightly different actual energy states.

   (b) Repeat your analysis for emission reactions producing one neutron from more and more excited states (these are the reactions that look like (n,n'15)). Using JANIS, graph the maximum cross section vs. the neutron's energy state, and the energy of non-zero cross section vs. excited state. What patterns do you see, and how can you explain the deviations you see in these patterns?
   Here is the graph of all the relevant cross sections:
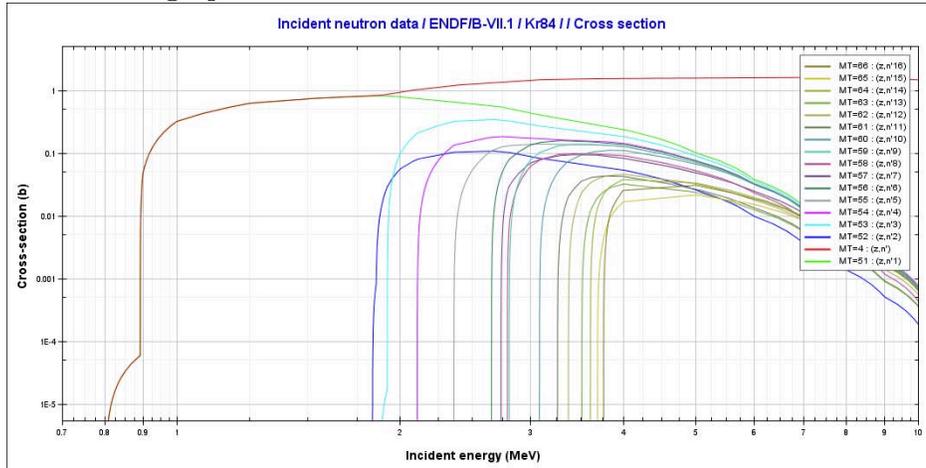
   The following table of values can be extracted from this data:

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[1] S. I. Sukhoruchkin, Z. N. Soroko. **Excited Nuclear States for Kr-84 (Krypton). Chapter Nuclei with Z = 30 - 47, Volume 25B of the series Landolt-Börnstein - Group I Elementary Particles, Nuclei and Atoms pp 1390-1399, Springer (2012).**

Excited State	E_{min} (Me)V	σ_{max} (b)
1	0.892	0.813
2	1.858	0.107
3	1.923	0.340
4	2.127	0.180
5	2.383	0.138
6	2.678	0.156
7	2.751	0.093
8	2.808	0.097
9	2.823	0.137
10	3.107	0.110
11	3.278	0.043
12	3.393	0.045
13	3.542	0.032
14	3.633	0.038
15	3.712	0.021
16	3.784	0.031
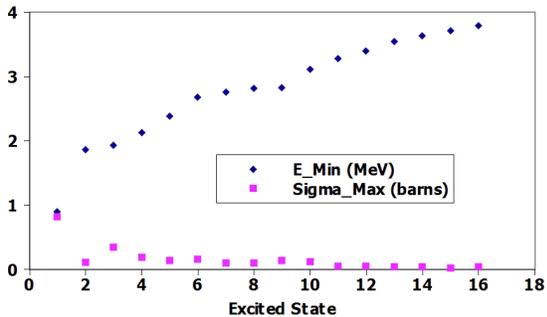
We can see that the energy required to eject more and more excited neutrons continues to increase, while the cross section for the same reaction decreases. This is because higher excitation states are less stable, being farther from the ground state.

4. Explain why delayed neutrons are the key to controlled neutron reactivity. Also explain why fast reactors are inherently more difficult to control.

   Without a small fraction of delayed neutrons, the chain reaction would proceed with a very, very fast period. Having a fraction of delayed neutrons allows one to have a critical reactor, while allowing some modicum of control. It also takes some time for the neutrons to slow down, meaning that fast reactors (which don't require the neutrons to slow down) are inherently faster feedback compared to thermal energy reactors.

2 Designing Effective Shielding

Your job is to design effective shielding for the MIT nuclear reactor, knowing that fission products decay with all manners of different mechanisms.

1. State which class of materials are best for shielding beta particles, gamma rays, x-rays, and neutrons, and state why using your knowledge of cross sections and stopping power.

   Just about everything shields beta particles, as their stopping power is by far the highest of all the particles mentioned here. They also originate in the fission products in the fuel, and the fuel itself provides adequate shielding. Gamma rays can only be effectively shielded by dense, high-Z materials. Therefore, lead and concrete are the best here: lead for its high-Z and inexpensiveness, concrete for its high density and great inexpensiveness. X-rays are more easily shielded than gamma rays, but proceed according to the same mechanisms: all three photon effect cross sections (photoelectric, Compton scattering, pair production) scale very strongly with increasing Z. Neutrons are all born fast inside the reactor, and therefore must be slowed down. Any hydrogen-dense material is most suitable for slowing down (moderating) the neutrons, at which point a strong thermal neutron absorber can consume them. Water is an excellent shield, as it has the highest moderating power and a non-negligible thermal capture cross section. Then, putting a strong absorber like boron in the way will absorb the neutrons. Plastics like RecoRad, a borated PVC, contain high densities of both hydrogen and boron to moderate and absorb, respectively.

2. What layers of shielding between the fuel and experimentalists outside the reactor will shield against all potential types of radiation, and in which order? Consider types of radiation produced when others interact with some of the layers of shielding. Estimate the required thicknesses of shielding in each case to remove 99.99% of the radiation emanating from the reactor.

Assuming that the water and concrete block almost all the fast neutrons from escaping, the only particles that will escape are high-energy gamma rays. The following shielding exists in the MIT reactor, intentionally or not, to block the following types of radiation:

Layer	Composition	Mostly Blocks	Reason	Produces
Fuel/Cladding	U/Al	α, β^-	High stopping power	x-rays (ionization by β^- & γ)
Coolant	H_2O	Fast neutrons	Excellent moderator	Not much
Reflector	D_2O	Fast neutrons	Good moderator and reflector	β^- from ^3H decay (neutron capture)
Shielding	Concrete	γ, x-rays	Thick, dense material	x-rays, mostly self-shielded

Assuming that only gamma rays escape the inner three layers, we can set up the simple attenuation equation to calculate the required thickness of concrete shielding to attenuate most of the gamma rays:

$$\frac{I}{I_0} = 0.0001 = e^{-\left(\frac{\mu}{\rho}\right)\rho x} \tag{7}$$

Using the NIST tables of x-ray attenuation coefficients for ordinary concrete, and assuming an average gamma ray energy of $1\,\text{MeV}$, and assuming a fully dense concrete density of $2.5\,\frac{g}{cm^3}$, we get an attenuation coefficient of $\left(\frac{\mu}{\rho}\right) \approx 0.1\,\frac{cm^2}{g}$. Solving for the concrete thickness, we get $x = 36.8\,\text{cm}$.. This assumes that attenuating 99.99% of the gammas brings the dose down to a reasonable level. From seeing the concrete at the MIT reactor, which is 1-2 meters thick, this is probably not the actual case.

3. Given your shielding array in the previous question, what would happen in terms of radiation exposure if the water in the MIT reactor were to boil away? Assume that the reactor continues producing neutrons for some time.

If the water were to boil away, the people in the reactor building would be subject to a *huge* dose from fast neutrons. Cross sections for all reactions for fast neutrons are very low, in the 0.1-10 barn region, meaning that many fast neutrons will escape the reactor. The concrete shielding, which contains *lots* of water in its structure (it is typically 15-20 percent water), will function as the primary neutron moderator in this case.

3 Applied Questions

For these questions, consider the MIT reactor in its critical state, and the various experiments that we do with it. Here is a cross section of the relevant parts of the MIT reactor:

What would be the effect of each of the following changes on the reactor's criticality, and which of the terms in the six factor formula would be affected? Explain why, using your knowledge of neutron absorption and leakage, and how they affect criticality.

For all these questions, we refer to the six factor formula:

$$\underbrace{k_{eff}}_{Multiplication\,factor} = \underbrace{\eta}_{Neutrons\,per\,fission}\underbrace{p}_{Resonance\,escape}\underbrace{f}_{Fuel\,utilization}\underbrace{\epsilon}_{Fast\,fission\,factor}\underbrace{P_{FNL}}_{Fast\,non-leakage}\underbrace{P_{TNL}}_{Thermal\,non-leakage} \tag{8}$$

1. Passing silicon ingots through the reactor, to dope them with phosphorus by transmutation
   Decreases resonance escape a little bit, by introducing an element with a higher cross section and more resonances than water (which has almost none) in place of water.
   May decrease fuel utilization a bit, by putting something new in the reactor.
   Decreases fast and thermal non-leakage, by replacing a good moderator with an OK one wherever the Si ingots are. Fast non-leakage is decreased more, as thermal neutrons don't leak out much anyway.
   Overall effect: Decrease k_{eff}

2. Inserting a small piece of stainless steel directly in the core of the reactor

    Decreases resonance escape. If stainless steel is put in place of water, then the overall reactor moderates less, making the spectrum a bit harder (faster).

    Decreases fuel utilization, because elements in the stainless steel (particularly nickel) have high absorption (n, γ) cross sections. This means more absorptions happen in places other than the fuel.

    Decreases fast non-leakage, by adding low-lethargy (ineffective) neutron moderator in the core. Does not change thermal non-leakage, as thermal neutrons in the center of the core will never make it to the outside.

    Overall effect: Decrease k_{eff}

3. Replacing the coolant with liquid sodium

    Greatly decreases resonance escape, as sodium is a *far* worse moderator compared to water. We neglect the change in sodium's capture cross section compared to water, as sodium's capture cross section is incredibly low.

    Can increase fuel utilization, because the lower absorption of sodium compared to water means fewer neutrons are absorbed in the coolant. May decrease fuel utilization, as more of the absorption events happen in the resonance regions of other materials (structural, cladding, etc.) because the neutrons stay faster (higher energy).

    Decreases fast non-leakage, as more neutrons stay faster and can scatter out to escape. Likely increases thermal non-leakage, as the overall, averaged thermal absorption cross section for the reactor is lower at about 0.025 eV (0.5 b) compared to water (1.1 b). Overall effect of non-leakage is stronger on fast neutrons, as there are more of them with sodium coolant.

    Overall effect: Decrease k_{eff}, assuming the MIT reactor geometry is held constant. One would need tighter fuel spacing and a reflector or something to keep the fast neutrons in and fully utilized.

4. Closing all the beamports which let neutrons out for experiments

    Increases fast and thermal non-leakage, by reflecting some of those neutrons back instead of letting them all leak out.

    Overall effect: Increase k_{eff}, but not by very much at all.

5. Raising the temperature of the coolant by 30 Kelvin

    The principal change here is the decrease in number density by the thermal expansion of everything in the core. The secondary effect is *spreading of the resonance regions* due to Doppler broadening, though this aspect wasn't discussed in class and won't be graded.

    Decreases resonance escape probability by Doppler broadening, but slightly increases resonance escape by lowering the Σ_a, specifically by lowering the number density.

    Decreases fuel utilization, because if the number density of the fuel decreases, more neutrons escape the core without hitting the fuel. A more complete answer would look at the change in total capture cross section resonance areas compared to fission cross section resonance areas, but that's a topic for 22.05!

    May change the fast fission factor, for the same reason noted in the line above.

    Decreases both fast and thermal non-leakage, by lowering the number density (and therefore total coolant mass) for a fixed geometry core.

    Overall effect: Decrease k_{eff}, because one *should* have negative temperature feedback coefficients!

6. Increasing the enrichment of the fuel

    Actually *decreases* neutrons per fission a bit *over time*, as there is less U-238 to produce Pu-239, which has a higher ν of about 2.9 compared to U-235's $\nu = 2.44$.

    Probably changes resonance escape, as U-235 and U-238 have different resonances and areas. Would have to do a complete resonance integral to figure this out. Then again, the fission cross sections also have resonances, which may *help* in this case.

    Greatly increases fuel utilization, as more absorption events directly result in fission.

Increases fast **fission** factor a bit, as U-238 has a lower fast **fission** cross section compared to U-235. Then again, the amount of Pu-239 in the core at any time will change this too, in a time-dependent way.

Decreases fast and thermal non-leakage, as capture & **fission** cross sections for U-235 are higher than U-238.

Overall **effect:** Increases k_{eff}, as generally higher **fission** to capture cross section ratio and more **fissile** vs. fertile fuel mean that more neutrons are created per neutron entering any atom in the core.

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