22.01 Fall 2015, Problem Set 1 Solutions

Due: September 18, 11:59PM on Stellar

September 19, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1 (50 points) Retracing Chadwick's Discovery of the Neutron

In thess questions, you will recreate some of James Chadwick's logic as he hypothesized and proved the existence of the neutron. Read the two papers on the Stellar site, "Possible Existence of a Neutron" and "The Existence of a Neutron," and answer the following questions.

1. What made James first hypothesize an uncharged particle with the mass of a proton?
James noticed that the radiation emanating from the bombardment of beryllium by alpha particles produced over 30,000 ion pairs, while a gamma ray of the expected energy would have only produced about 10,000 ion pairs. Therefore, if a gamma ray was responsible, it was too high in energy, which would have violated energy conservation.

2. What was the competing hypothesis to explain the observed results?
The competing hypothesis was that a more massive, but still uncharged, particle must be responsible for the ion pairs.

3. Write the nuclear reaction of alpha particles (helium nuclei) bombarding beryllium. You may want to look up the stable isotope of Be here: http://atom.kaeri.re.kr/
The reaction of alpha particles bombarding beryllium is as follows:

$$^4_2He +^9_4 Be \rightarrow^1_0 n +^{12}_6 C \tag{1}$$

Of course now we know that this is the correct reaction. The competing (and incorrect) hypothesis would have been the creation of carbon-13:

$$^4_2He +^9_4 Be \rightarrow^{13}_6 C + \gamma \tag{2}$$

4. Why would a neutron have greater "penetrating power" (range) through matter compared to charged particles? What does a neutron not interact with?
The neutron should have a greater penetrating power (range) because it is uncharged. It therefore does not strongly interact with the electrons in the nucleus.

5. On p. 694 of the second paper, Chadwick states that "The source of polonium was prepared from a solution of radium by deposition on a disc of silver." How could polonium be produced directly from radium?
Looking at the KAERI Table of Nuclides, one can see that the longest-lived isotope of Radium is ^{226}Ra***, which has a half life of 1600 years. It decays by alpha decay to*** ^{222}Ra***, which itself decays to*** ^{218}Po***. Polonium has quite a short half life, which means that almost as soon as it's made, it emits its characteristic alpha particle.***

6. On p. 698 of the second paper, Chadwick states that "the mass of the neutron is equal to that of the proton..." Is this true? What are the masses of the proton, neutron, and electron? Is the mass of Rutherford's "neutron," consisting of a proton and an electron, equal to the neutron's mass? Why or why not (where does the energy discrepancy come from)? Why couldn't Chadwick discern between the masses of these two particles?
The masses of the proton and the neutron are not equal, though they are very close, so it's conceivable that back in the 1930's, measuring the differences in mass would have been too difficult. The masses are as follows, as sourced from NIST (http://physics.nist.gov/cgi-bin/cuu/Value?mesmp|search_for=atomnuc!):
Neutron mass: $1.674927471 \cdot 10^{-27}$ kg
Proton mass: $1.672621898 \cdot 10^{-27}$ kg
Electron mass:$9.10938356 \cdot 10^{-31}$ kg
This leaves a mass discrepancy of $6.80381056 \cdot 10^{-31}$ kg***, almost 3/4 of the mass of an electron. This extra energy comes from the conversion of some of the mass of the neutron to kinetic energy of the proton and electron leaving the reaction.***

7. On pp. 701-702, why is the kinetic energy of ^{11}B not accounted for, and what does it mean for kinetic energies to be given in "mass units?" Convert these "mass unit" energies to energies in electron volts (eV). What is the approximate kinetic energy of ^{11}B in eV at room temperature?
The kinetic energy of ^{11}B ***is ignored, because it is so very, very small compared to the MeV energies involved in nuclear reactions. The value of this kinetic energy can be found by multiplying Boltzmann's constant*** $\left(8.6173324 \cdot 10^{-5} \frac{eV}{K}\right)$ ***by room temperature*** $(298\,K)$***, we get 0.025 eV. Giving energies in mass units means that we are equating mass and energy by Einstein's relation:***
$$E = mc^2 \tag{3}$$

2 (50 points) Getting Used to Nuclear Quantities

In these questions, you will calculate a number of quantities related to nuclear reactions and power generation. You will have to look up certain reactions and values from *primary sources* in the literature (books, papers, databases). Make sure to state which values you look up or assume, and *cite your sources* using proper citation methods.

These calculations are useful, especially when arguing the benefits and costs of nuclear power. If you can derive them quickly and by yourselves, you don't have to rely on as many other sources of information to make your point.

2.1 Relative Power Densities

Calculate the energy released from burning 1kg of coal, natural gas, uranium, and deuterium. Now repeat this calculation for the nuclear fission of uranium, and the nuclear fusion of deuterium.
The equations for the five reactions asked for are as follows:

$$C + O_2 \rightarrow CO_2 + E_1 \tag{4}$$

$$CH_4 + 2O_2 \rightarrow CO_2 + H_2O + E_2 \tag{5}$$

$$U + O_2 \rightarrow UO_2 + E_3 \tag{6}$$

$$^{235}_{92}U +^1_0 n \rightarrow FP_1 + FP_2 + 2.44\left(^1_0 n\right) + \gamma + E_4 \tag{7}$$

$$^2_1H +^2_1 H \rightarrow^3_1 He +^1_0 n + E_5 \qquad or \qquad ^2_1H +^2_1 H \rightarrow^3_1 H +^1_0 p^+ + E_5 \tag{8}$$

For Equation 4, we may first assume that coal is not 100% carbon, according to the Energy Information Administration (EIA) ranges from 60% for lignite to 80% from anthracite[1]. Let's assume that it's 75%

[1] http://www.eia.gov/coal/production/quarterly/co2_article/co2.html

Energy	Energy	Unit	eV/atom	Molar Mass $\left(\frac{g}{mol}\right)$	Efficiency	J/kg
E_1	393.5	$\frac{kJ}{mol}$	4.1	12.011	0.75	$2.458 \cdot 10^7$
E_2	604.7	$\frac{kJ}{mol}$	6.3	16.043	1	$3.770 \cdot 10^7$
E_3	1,085	$\frac{kJ}{mol}$	11.3	235	1	$4.616 \cdot 10^6$
E_4	191.6	MeV	$1.91 \cdot 10^8$	235	1	$7.803 \cdot 10^{13}$
E_5	3.268	MeV	$3.267 \cdot 10^6$	4	1	$7.841 \cdot 10^{13}$

Table 2: Final energies in Problem 2.1

carbon. We can also assume that methane and uranium don't incur efficiency factors for "burning," and by that we mean in the chemical sense. For equation 7, we should assume two fission products at random, as long as they maintain mass conservation.

First, we can use the CRC Handbook from the MIT libraries site to look up chemical binding energies in $\frac{kJ}{mol}$[2] for equations 4-6. For equations 7-8, we can use the KAERI table of nuclides to look up the binding energies of the nuclei involved, a direct analogue to the chemical binding energies. Remember that the chemical binding energies of pure elements, as well as the nuclear binding energies of lone nucleons, are zero. Table 1 shows the binding energies from the two sources. Using these values, we can calculate the missing energies in Equations 4-8, shown in Table 2. These energies must then be all converted to the same value for direct comparison, let's use $\frac{eV}{atom}$ as the unit. The MeV energies are easy, just divide by 1,000,000. For $\frac{kJ}{mol}$, we use the following equation:

$$E\left[\frac{kJ}{mol}\right] * \left[\frac{1\,mol}{6 \cdot 10^{23}\,atoms}\right] * \left[\frac{1\,eV}{1.6 \cdot 10^{-22}\,kJ}\right] = E\left[\frac{eV}{atom}\right] \tag{9}$$

Finally, we convert these energies in $\frac{eV}{atom}$ to $\frac{J}{kg}$ as follows:

$$E\left[\frac{eV}{atom}\right] * \left[\frac{6 \cdot 10^{23}\,atoms}{1\,mol}\right] * \left[\frac{1.6 \cdot 10^{-19}\,J}{1\,eV}\right] * \left[\frac{1\,mol}{< Mol\,Mass > g}\right] * [Efficiency] * \left[\frac{1000\,g}{1\,kg}\right] = E\left[\frac{J}{kg}\right] \tag{10}$$

Species	Binding Energy	Unit
CO_2 (g)	393.5	$\frac{kj}{mol}$
CH_4 (g)	74.6	$\frac{kj}{mol}$
H_2O (g)	285.8	$\frac{kj}{mol}$
UO_2 (s)	1,085	$\frac{kj}{mol}$
^{235}U	1,784	MeV
^{90}Sr	782.6	MeV
^{145}Gd	1,193	MeV
^{2}H	2.225	MeV
^{3}He	7.718	MeV
^{3}H	8.482	MeV

Table 1: Binding energies for Problem 2.1

2.2 Accelerator Energetics

A common tool to provide data on nuclear reactions is the electrostatic accelerator. These work by accelerating charged particles through a large, static electric field. We consider here an accelerator that provides a 2MV potential drop over 2m for double charged nickel ions $\left(Ni^{+2}\right)$, which enter the accelerator at ˜zero kinetic energy into the accelerator from an ion source.

2.2.1 What will be the kinetic energy of a nickel ion in eV, as it exits the accelerator?

The energy imparted by an accelerator is equal to the charge of the particle being accelerated times the voltage, so this equals

$$E_{kinetic} = 2\left(1.6 \cdot 10^{-19}\,C\right) * \left(2 \cdot 10^6\,V\right) = 6.4 \cdot 10^{-19}\,J = 4\,MeV \tag{11}$$

2.2.2 What will be its *total mass* (not its rest mass) as it exits the accelerator?

The accelerator transferred 4 MeV of energy to the Ni ion, imparting kinetic energy. First, we find the rest mass energy of the Ni ion. Let's choose a specific isotope, Ni-58, for this calculation, which has a mass of 57.94 amu:

$$E_{rest\,mass} = m_0 c^2 = 54,475\,MeV \tag{12}$$

[2] http://www.hbcpnetbase.com/

Checking our math for sanity, this is just about 58 times the rest mass of the proton (938 MeV). Then, we know the equation for the total energy is:

$$E_{total} = \gamma m_0 c^2 = E_{rest\,mass} + E_{kinetic} \tag{13}$$

Using this equation, we find the value of γ to be 1.00007, which is *waaay* not relativistic. Its total mass is just as follows:

$$m_{total} = m_0 \gamma = 9.699 \cdot 10^{-26}\, kg \tag{14}$$

which is just 1.00007 times its rest mass.

2.2.3 If the ion source injects 2mA of current, what is the total number of particles leaving the accelerator per second?

2mA of current represents the motion of 0.002 Coulombs per second. Each ion is *doubly charged*, so each has a charge of $3.2 \cdot 10^{-19}$ C. To find the number of particles, we convert as follows:

$$\frac{\#\, ions}{sec} = 0.002\, \frac{C}{sec} * \frac{1\, ion}{3.2 \cdot 10^{-19}\, C} = 6.25 \cdot 10^{15}\, \frac{ions}{sec} \tag{15}$$

2.2.4 What is the total power, in Watts, associated with pulling 2mA of current through 2MV of electrostatic potential? Where does this power go?

The power in a beam is the same as through any circuit:

$$P = IV = (0.002\, A)\left(2 \cdot 10^6\, V\right) = 4\, kW \tag{16}$$

This energy must be dissipated in the target that it hits, usually as heat, which must be removed.

2.3 Mass-Energy Equivalence

From general relativity, the mass (m) of a moving particle can be expressed as follows:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{17}$$

where m_0 is its rest mass, v is its velocity, and c is the speed of light.

2.3.1 What is the particle's mass at the following speeds: 1 $\frac{m}{s}$, 1 $\frac{km}{s}$, 1 $\frac{Mm}{s}$, 0.9c, 0.99c, c?

Using Equation 17, and expressing the mass in terms of m_0, we have the following masses:

$$v = 1\, \frac{m}{s} \Rightarrow m = m_0\, (to\, floating\, point\, precision) \tag{18}$$

$$v = 1\, \frac{km}{s} \Rightarrow m = 1.00000000001 m_0 \tag{19}$$

$$v = 1\, \frac{Mm}{s} \Rightarrow m = 1.0000056 m_0 \tag{20}$$

$$v = 0.9c \Rightarrow m = 2.294 m_0 \tag{21}$$

$$v = 0.99c \Rightarrow m = 7.089 m_0 \tag{22}$$

$$v = c \Rightarrow m = \infty \tag{23}$$

2.3.2 Derive an expression for the particle's kinetic energy (T) in terms of its total and rest masses.

The total energy of a particle is given in terms of its rest mass energy and kinetic energy:

$$E_{total} = T + E_{rest\,mass} = \gamma m_0 c^2 \tag{24}$$

and we know that the rest mass energy is given by $\mathrm{E_{rest\,mass} = m_0c^2}$, therefore:

$$T = (\gamma - 1)\, m_0 c^2 \tag{25}$$

2.3.3 Show that the particle's momentum (p) can be described in terms of its kinetic energy and rest mass as follows:

$$p = \frac{1}{c}\sqrt{T^2 + 2Tm_0c^2} \tag{26}$$

We can start with the expression for total relativistic energy of a particle:

$$E = \sqrt{p^2c^2 + E_{rest\,mass}} = \sqrt{p^2c^2 + m_0^2c^4} \tag{27}$$

We can then square each side, and isolate the term containing the momentum:

$$p^2c^2 = m^2c^4 - m_0^2c^4 \tag{28}$$

Now we recognize that the total mass (m) is related to the rest mass ($\mathrm{m_0}$) by the factor γ:

$$p^2c^2 = m_0^2\gamma^2c^4 - m_0^2c^4 \tag{29}$$

Then we can use the relation in Equation 25 for the kinetic energy:

$$\frac{T}{m_0c^2} = (\gamma - 1) \Rightarrow \gamma = \frac{T}{m_0c^2} + 1 \tag{30}$$

We then plug this into Equation 29:

$$p^2c^2 = m_0^2c^4\left[\left[\frac{T}{m_0c^2} + 1\right]^2 - 1\right] \tag{31}$$

Factoring this out, we get:

$$p^2c^2 = m_0^2c^4\left[\frac{T^2}{m_0^2c^4} + \frac{2T}{m_0c^2} + 1\right] - 1 = T^2 + 2Tm_0c^2 \tag{32}$$

Now we just take the square root of each side and divide by the speed of light:

$$pc = \sqrt{T^2 + 2Tm_0c^2} \Rightarrow p = \frac{1}{c}\sqrt{T^2 + 2Tm_0c^2} \tag{33}$$

2.3.4 At what ratio of kinetic energy to rest mass energy is the classical (non-relativistic) expression for momentum accurate to one part per thousand?

Here we are looking for the ratio of kinetic energy to rest mass energy which gives a relativistic momentum equal to 1.001 times the classical momentum:

$$\frac{p_{relativistic}}{p_{classical}} = \frac{m_0\gamma v}{m_0 v} = \gamma = 1.001 \tag{34}$$

All we have to do is find the ratio of kinetic to rest mass energies when $\gamma = 1.001$:

$$\frac{E_{kinetic}}{E_{rest\,mass}} = \frac{(\gamma - 1)\, m_0c^2}{m_0c^2} = (\gamma - 1) = 0.001 \tag{35}$$