1. Solution:

$$g_1(x,z) = a_1 \exp\left\{i2\pi\left(\frac{x}{\lambda}\sin\frac{\pi}{6} + \frac{z}{\lambda}\cos\frac{\pi}{6}\right)\right\}, \qquad a_1 = 1$$
$$g_2(x,z) = a_2 \exp\left\{i2\pi\left(-\frac{x}{\lambda}\sin\frac{\pi}{3} + \frac{z}{\lambda}\cos\frac{\pi}{3}\right)\right\}, \qquad a_2 = \frac{1}{2}$$

$$\begin{aligned}\text{At } z=0, \quad I(x) &= |g_1(x,0)+g_2(x,0)|^2 = \left|a_1 e^{i2\pi\frac{x}{\lambda}\cdot\frac{1}{2}} + a_2 e^{-i2\pi\frac{x}{\lambda}\cdot\frac{\sqrt{3}}{2}}\right|^2 \\ &= (a_1^2+a_2^2)\left[1+\frac{2a_1a_2}{a_1^2+a_2^2}\cos\left(2\pi\frac{x}{\lambda}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right)\right] \\ &= \left(1+\frac{1}{4}\right)\left[1+\frac{2\times 1\times\frac{1}{2}}{1+\frac{1}{4}}\cos\left(2\pi\frac{x}{\lambda}\cdot\frac{1+\sqrt{3}}{2}\right)\right] \\ &= \frac{5}{4}\left[1+\frac{4}{5}\cos\left(2\pi\frac{x}{\Lambda}\right)\right] \quad \text{where } \Lambda = \frac{2\lambda}{1+\sqrt{3}}\end{aligned}$$

$$m = \frac{4}{5} \rightarrow \text{ contrast} \qquad \Lambda = \frac{2\lambda}{1+\sqrt{3}} \rightarrow \text{ period}$$
$$I(0) = \frac{5}{4}\times\left(1+\frac{4}{5}\right) = \frac{5}{4}\times\frac{9}{5} = \frac{9}{4} = 2\frac{1}{4}$$

After phase-shifting g_1 by $\pi/2$, we get

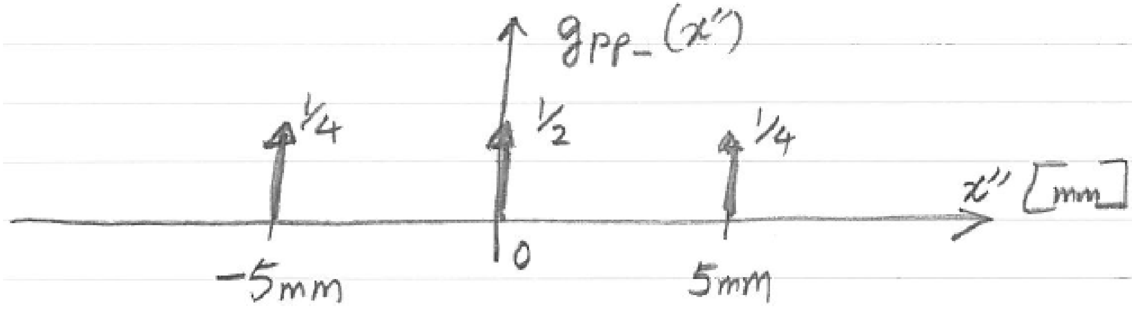
$$I(x) = \frac{5}{4}\left[1+\frac{4}{5}\cos\left(2\pi\frac{x}{\Lambda}+\frac{\pi}{2}\right)\right] \Rightarrow I(0) = \frac{5}{4} = 1\frac{1}{4}$$

2. (a) Solution:

$$\begin{aligned}g_t(x) &= \frac{1}{2}\left[1+\cos\left(2\pi\frac{x}{\Lambda}\right)\right] \\ &= \frac{1}{2}\left[1+\frac{1}{2}e^{i2\pi\frac{x}{\Lambda}}+\frac{1}{2}e^{-i2\pi\frac{x}{\Lambda}}\right]\end{aligned}$$

Fourier:
$$G_t(u) = \frac{1}{2}\left[\delta(u)+\frac{1}{2}\delta\left(u-\frac{1}{\Lambda}\right)+\frac{1}{2}\delta\left(u+\frac{1}{\Lambda}\right)\right]$$

Scaling:
$$\begin{aligned}g_{PP-}(x'') &\propto G_t\left(\frac{x''}{\lambda f}\right) \\ &= \frac{1}{2}\delta(x'')+\frac{1}{4}\delta\left(x''-\frac{\lambda f}{\Lambda}\right)+\frac{1}{4}\delta\left(x''+\frac{\lambda f}{\Lambda}\right)\end{aligned}$$
$$\frac{\lambda f}{\Lambda} = \frac{1\mu\text{m}\times 5\text{cm}}{10\mu\text{m}} = 5\text{mm}$$



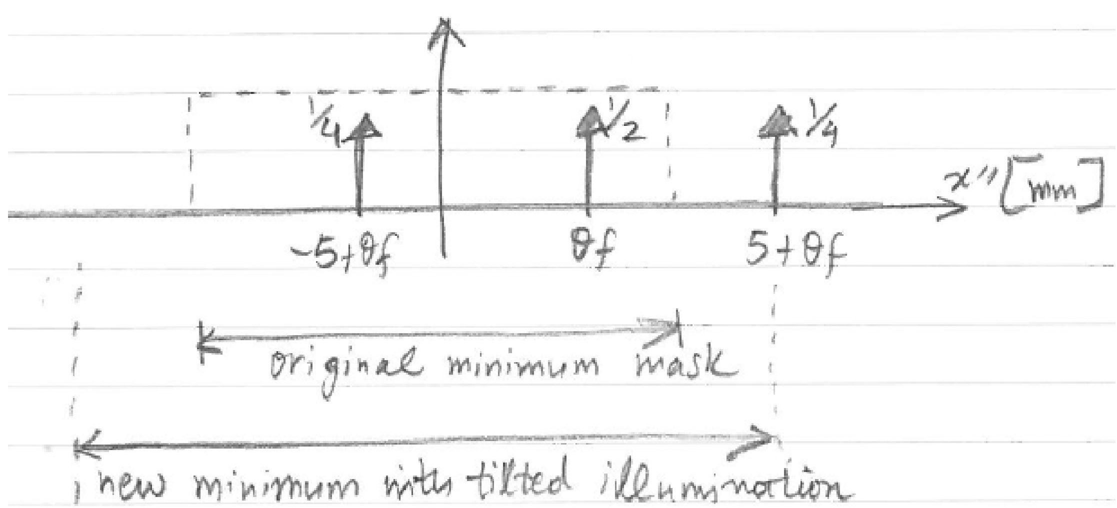
(b) Solution: To preserve all the input's details to the outputs, all diffracted orders must pass the pupil mask. $\therefore a > 1\text{cm}$.

(c) Solution: Yes. If the illumination is $e^{i2\pi\theta\frac{x}{\lambda}}$ then:

$$\begin{aligned}
g_{\text{in}}(x) &= g_{\text{illum}}(x) g_t(x) \\
&= \frac{1}{2}\left[e^{i2\pi\theta\frac{x}{\lambda}} + \frac{1}{2}e^{i2\pi(\frac{\theta}{\lambda}+\frac{1}{\Lambda})x} + \frac{1}{2}e^{i2\pi(\frac{\theta}{\lambda}-\frac{1}{\Lambda})x}\right] \\
G_{\text{in}}(u) &= \frac{1}{2}\delta\left(u-\frac{\theta}{\lambda}\right) + \frac{1}{4}\delta\left(u-\frac{\theta}{\lambda}-\frac{1}{\Lambda}\right) + \frac{1}{4}\delta\left(u-\frac{\theta}{\lambda}+\frac{1}{\Lambda}\right) \\
g_{PP-}(x'') &\propto \frac{1}{2}\delta(x''-f\theta) + \frac{1}{4}\delta\left(x''-\theta f-\frac{\lambda f}{\Lambda}\right) + \frac{1}{4}\delta\left(x''-\theta f+\frac{\lambda f}{\Lambda}\right)
\end{aligned}$$

So all diffracted orders shift by θf at the pupil plane.

The original limit $a_{\min} = 1\text{cm}$ is not enough to accommodate all orders. A pupil mask that is symmetric around the optical axis must be extended to $\frac{\lambda f}{\Lambda} + \theta f$:



(d) Solution: The glass plate shifts the 0th order by π:

$$g_{PP-}(x'') = \frac{1}{2}\delta(x'') + \frac{1}{4}\delta(x''-5) + \frac{1}{4}\delta(x''+5)$$

$$g_{PP+}(x'') = \underbrace{-\frac{1}{2}\delta(x'')}_{\substack{\text{phase}\\\text{shift}\\\text{by }\pi}} + \underbrace{\frac{1}{4}\delta(x''-5) + \frac{1}{4}\delta(x''+5)}_{\text{no change}}$$

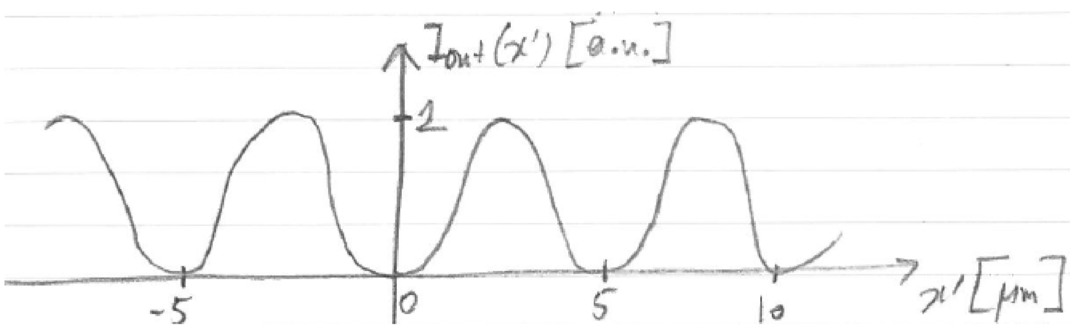
Fourier: $$G_{PP+}(u) = -\frac{1}{2} + \frac{1}{4}e^{i2\pi u\cdot 5\text{mm}} + \frac{1}{4}e^{-i2\pi u\cdot 5\text{mm}}$$

Scaling: $$g_{\text{out}}(x') = G_{PP+}\left(\frac{x'}{\lambda f}\right) = -\frac{1}{2} + \frac{1}{4}e^{i2\pi\frac{x'}{10\mu\text{m}}} + \frac{1}{4}e^{-i2\pi\frac{x'}{10\mu\text{m}}}$$

$$= -\frac{1}{2} + \frac{1}{2}\cos\left(2\pi\frac{x'}{10\mu\text{m}}\right) = -\frac{1}{2}\left[1 - \cos\left(2\pi\frac{x'}{10\mu\text{m}}\right)\right]$$

$$I_{\text{out}}(x') = \left|g_{\text{out}}(x')\right|^2 = \frac{1}{4}\left[1 - \cos\left(2\pi\frac{x'}{10\mu\text{m}}\right)\right]^2$$

$$= \frac{1}{4}\left[2\sin\left(2\pi\frac{2x'}{10\mu\text{m}}\right)\right]^2 = \sin^2\left(2\pi\frac{2x'}{10\mu\text{m}}\right)$$



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