2.58 HW5 Solutions

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Prob 10.1

According to eq.(10.19), for a harmonic oscillator,

$$\Delta E = h \nu_e \Delta v$$

$$\Rightarrow \quad \nu_e = \frac{\Delta E}{h \Delta v_e} = \frac{h c_0 \eta}{h \Delta v} = \frac{c_0}{\Delta v} \eta$$

From table 10.3, we have

$$\nu_e = \frac{c_0}{\Delta v} \eta \approx \frac{3 \times 10^{10}}{1} \times 2143 = 6.429 \times 10^{13}\ Hz$$

$$\approx \frac{3 \times 10^{10}}{2} \times 4260 = 6.390 \times 10^{13}\ Hz$$

Prob 10.7

(a) The Elsasser model.

The unit of the line strength (S) suggests that a mass absorption coefficient has been used.

At 500 K and 1atm,

$$\rho = \rho_{STP} \cdot \frac{T_{STP}}{T} = 3 \times 10^{-3} \times \frac{273}{500} = 1.638 \times 10^{-3}\ g/cm^3$$

$$X = \rho s = 1.638 \times 10^{-3}\ g/cm^3 \times 50\ cm = 8.19 \times 10^{-2}\ g/cm^2$$

$$x = \frac{S X}{2\pi b_L} = \frac{2.04 \times 10^{-4}\ cm^{-1}/(g/m^2) \times 8.19 \times 10^{-2}}{2\pi \times 0.04\ cm^{-1}} = 2.09/\pi$$

$$\beta = \frac{\pi b_L}{d} = \frac{\pi \times 0.04}{0.25} = 0.16\pi$$

$$\tau = 2\beta x = 0.669$$

According to (10.38),

$$L(x) = x\left[1 + \left(\frac{\pi x}{2}\right)^{5/4}\right]^{-2/5} = 0.499$$

$$\bar{\varepsilon}_\eta = \operatorname{erf}\left(\sqrt{\pi}\, \beta L(x)\right) = \operatorname{erf}\left(\sqrt{\pi} \times 0.16\pi \times 0.499\right) = 0.471$$

Prob 10.21

(a) For simplicity, we will assume a constant average pressure of 0.5 atm for the atmosphere (see prob. 10.20 or apply Eqns (10.129-131) for more accurate results).

I believe this is the correct form. Nevertheless, I didn't take any point because you used another form.

$$P_e = \left[\frac{P}{P_0}\left(1+(b-1)\frac{P_a}{P}\right)\right]^n = \left[0.5\left(1+0.12\frac{10^{-6}}{0.5}\right)\right]^{0.6} \simeq 0.660$$

$$X_1 = P_a \cdot L_1 = 1\times10^{-6}\,\text{atm} \times 1\times10^{5}\,\text{cm} = 0.1\ \text{cm}\cdot\text{atm}$$

$$\beta_1 = \gamma_1 P_e = 0.145\times0.660 = 0.0957$$

$$\beta_2 = \gamma_2 P_e = 0.377\times0.660 = 0.249$$

$$\tau_{01} = \alpha_1 X_1/\omega_1 = \frac{2035\ \text{cm}^{-2}\text{atm}^{-1}\times 0.1\ \text{cm}\cdot\text{atm}}{22\ \text{cm}^{-1}} = 9.25$$

$$A_1^* = 2\sqrt{\tau_{01}\beta_1} - \beta_1 = 2\sqrt{9.25\times0.0957} - 0.0957 = 1.786$$

$$A_1 = A_1^*\omega_1 = 39.29\ \text{cm}^{-2}$$

$$\tau_{02} = \frac{\alpha_2 X_2}{\omega_2} = \frac{161\times1\times10^{-6}}{18.5}L_2 = 8.703\,L_2\times1\times10^{-6}$$

By trial and error, we know $1/\beta < \tau_0 < \infty$

$$A_2^* = \ln(\tau_{02}\beta_2)+2-\beta = \ln(8.703\times10^{-6}\times0.249\,L_2)+2-0.249$$

$A_1 = A_2 \Rightarrow$

$$39.29 = 18.5\times\left[\ln(2.167\times10^{-6}L_2)+1.751\right]$$

$$\Rightarrow L_2 \simeq 6.7\times10^{5}\ \text{cm} = 6.7\ \text{km}$$

(b) ~~Assuming small change of L_1 so that the above correlations remain valid for A_1^* and A_2^*.~~

All the empirical correlations listed in Table 10.2 are monotone increasing with τ_0 ($\tau_0 = \frac{\alpha P}{\omega}\cdot L$). By requiring $A_1 = A_2$, we know L_2 will decrease if L_1 was decreased.

Also, A plot can show L_2/L_1 will decrease if L_1 was down. 2

Prob 10.29

According to Eq.(10.138),

$$\varepsilon = \sum_i^N \left(\frac{E_{b\eta_0}}{T^3}\cdot\frac{\omega}{\sigma T}\right)_i A_i^*$$

Compare $\left(\frac{\alpha E_{b\eta_0}}{T^3}\right)_i$ for all the bands, we find that only the 15 μm and 4.3 μm bands are important for CO_2.

The partial pressure of CO_2 is given by:

$$\rho_a = \frac{MP}{R_u T}y = \frac{44 g/mol \times 0.75 \times 1.01 \times 10^5 Pa}{8.3144 J/molK \times 600K} = 668.1y \ [g/m^3]$$

Where y is the concentration percentage of CO_2.

Use wbmco2cl.exe in Appendix F to yield

	ψ^*/ψ_0	ϕ/ϕ_0
15 μm	1.0	2.82133
4.3 μm	1.0	2.44723

$\Rightarrow$

	α_i	ω_i	γ_i
15 μm	19.0	31.11	0.428
4.3 μm	110.0	27.43	1.481

(a) 0.01% CO_2

$$P_{e_1} = \left[\frac{P}{P_0}\left(1+(b-1)\frac{P_a}{P}\right)\right]^n = \left[\frac{0.75}{1}\left(1+0.3\times 1\times 10^{-4}\right)\right]^{0.7} = 0.818$$

$$P_{e_2} = 0.794$$

$$X_1 = X_2 = \rho_a L = 0.0668, \quad \beta_1 = \gamma_1 P_{e_1} = 0.350, \ \beta_2 = 1.176$$

$$\tau_{01} = \frac{\alpha_1 X_1}{\omega_1} = 0.0408, \quad \tau_{02} = 0.268$$

$$\Rightarrow A_1^* = 0.0408, \quad A_2^* = 0.268 \quad \text{(linear regime)}$$

$$\varepsilon = \left(\frac{E_{b\eta_1}}{T^3}\cdot\frac{\omega_1}{\sigma T}\right)A_1^* + \left(\frac{E_{b\eta_2}}{T^3}\cdot\frac{\omega_2}{\sigma T}\right)A_2^*$$

$$\mathcal{E} = 1.30013\times10^{-8}\,\frac{31.11}{5.67\times10^{-8}\times600}\times0.0408 + 0.82748\times10^{-8}\times\frac{27.43\times0.268}{5.67\times10^{-8}\times600}$$

$$= 2.27\times10^{-3}$$

For (b) and (c), we can repeat the same procedures as in (a) to obtain the emissivity. The results are tabulated below

CO_2 concentration	$\mathcal{E}$ (wide band model)	$\mathcal{E}$ (Leckner's model)
0.01%	0.00227	0.00308
1%	0.0574	0.0431
100%	0.139	0.150