13.022
SPRING 98

QUIZ #2

SOLUTIONS

1.

$\lambda \gg a$
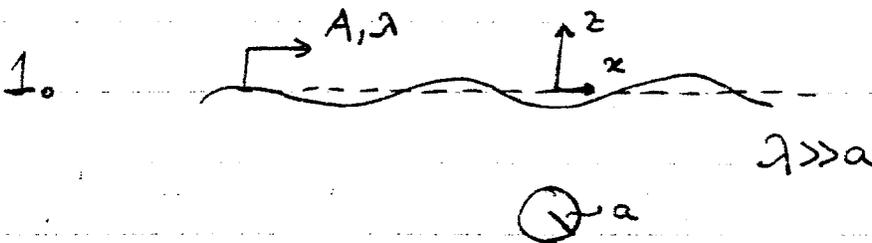
a)

- USE G I TAYLOR TO DETERMINE EXCITING FORCES

SURGE: $\quad X_1(t) = \left(a_{11} + \rho \forall\right) \dfrac{\partial^2 \phi_I}{\partial x\, \partial t}$

HEAVE: $\quad X_3(t) = \left(a_{11} + \rho \forall\right) \dfrac{\partial^2 \phi_I}{\partial z\, \partial t}$

$a_{11} = \rho \forall = \pi \rho a^2$

EQUATIONS OF MOTION ω. NO DAMPING + RESTORING

$$\left(a_{11} + \rho \forall\right) \ddot{X}_1(t) = X_1(t) = \left(a_{11} + \rho \forall\right) \dfrac{\partial^2 \phi_I}{\partial t\, \partial x}$$

$$\left(a_{11} + \rho \forall\right) \ddot{X}_3(t) = X_3(t) = \left(a_{11} + \rho \forall\right) \dfrac{\partial^2 \phi_I}{\partial t\, \partial z}$$

$$\Rightarrow \quad \ddot{X}_1 = \frac{\partial^2 \phi_I}{\partial t \, \partial x} \Rightarrow \text{SURGE ACCELERATION} \equiv$$

FLUID ACCELERATION $\equiv$

PARTICLE ACCELERATION AT

LOCATION OF CYLINDER

$$\ddot{X}_3 = \frac{\partial^2 \phi_I}{\partial t \, \partial z} = \quad \text{—} \, \text{''} \, \text{—}$$

THUS CIRCLE ACCERATION IN HEAVE + SURGE $\equiv$
PARTICLE ACCELERATION AT ALL TIMES t. THUS
TRAJECTORY OF CIRCLE SAME AS TRAJECTORN OF
PARTICLES —

b) INTRODUCE DAMPING:

SURGE :

$$(a_{11} + \rho \forall) \ddot{X}_1 + B \dot{X}_1 = (a_{11} + \rho \forall) \frac{\partial^2 \phi_I}{\partial t \, \partial x}$$

LET $X_1(t) = \text{Re} \{ \mathbb{X}_1 \, e^{i\omega t} \}$

$$\frac{\partial^2 \phi_I}{\partial x \, \partial t} = \text{Re} \left\{ \frac{i g A}{\omega} (i\omega)(-i\nu) \, e^{\nu z - i\nu x + i\omega t} \right\}$$

$$\Rightarrow \left[2\pi\rho a^2 (-\omega^2) + i\omega B \right] \mathbb{X}_1 = i g A \, \nu \, e^{-\nu d}$$

$$\Rightarrow \mathbb{X}_1 = \frac{i g A \, \nu \, e^{-\nu d}}{-\omega^2 (2\pi\rho a^2) + i\omega B} = \frac{i \omega^2 A \, e^{-\nu d}}{(i\omega)^2 \, 2\pi\rho a^2 + i\omega B}$$

$$\Xi_1 = \frac{\omega A\, e^{-\nu d}}{B + 2\pi \rho a^2 (i\omega)} = \frac{\omega A\, e^{-\nu d}}{B^2 + (2\pi \rho a^2 \omega)^2} [B - 2\pi \rho a^2 i\omega$$

$$\equiv |\Xi_1| e^{i\phi_1}$$

$$\Rightarrow \quad |\Xi_1| = \frac{\omega A\, e^{-\nu d}}{[B^2 + 2\pi \rho a^2 \omega^2]^{1/2}}$$

$$\tan \phi_1 = \frac{-2\pi \rho a^2 \omega}{B} \Rightarrow \phi_1 = \tan^{-1}\left[\frac{-2\pi \rho a^2 \omega}{B}\right]. \text{—}$$

HEAVE

$$\frac{\partial^2 \phi_I}{\partial t \partial z} = \text{Re}\left\{ \frac{iSA}{\omega} (i\omega) \nu\, e^{\nu z - i\nu x + i\omega t} \right\}$$

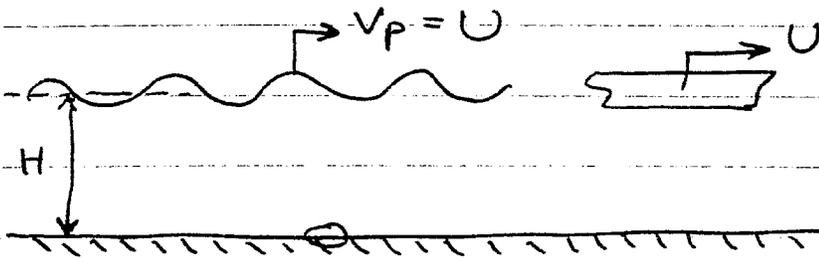$$= \text{Re}\left\{ -\omega^2 A\, e^{\nu z - i\nu x + i\omega t} \right\}$$

$$\Rightarrow \quad \text{BY ANALOGY TO SURGE:}$$

$$\Xi_3 = \frac{i\omega A\, e^{-\nu d}}{B + (2\pi \rho a^2) i\omega} = |\Xi_3| e^{i\phi_3}$$

$$|\Xi_3| = \frac{\omega A\, e^{-\nu d}}{[B^2 + 2\pi \rho a^2 \omega^2]^{1/2}}$$

$$\phi_3 = \tan^{-1}\left[\frac{B}{2\pi \rho a^2 \omega}\right]. \text{—}$$

2.

- PHASE VELOCITY $V_P = U = \frac{\omega}{K} \Rightarrow \omega = KU$

- DISPERSION RELATION : $\omega^2 = g\,k\,\tanh kH$

$$\Rightarrow \frac{\omega^2}{g} H = KH\,\tanh KH$$

$$\Rightarrow (KU)^2 \frac{H}{g} = KH\,\tanh KH,$$

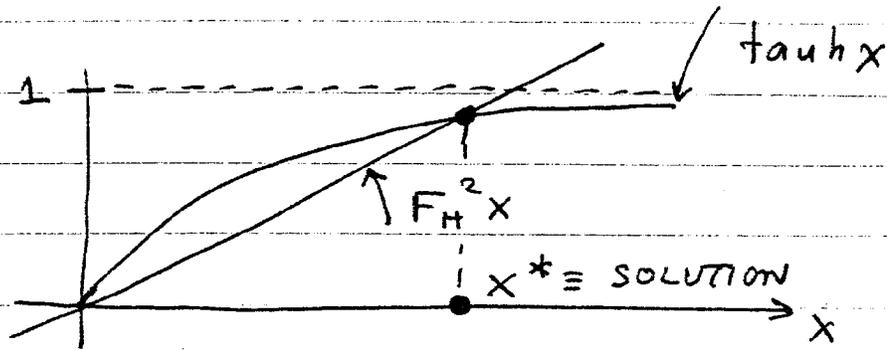$$\Rightarrow \frac{U^2}{g} K = \tanh kH \Rightarrow \frac{U^2}{gH}(kH) = \tanh(kH)$$

LET $KH = X$ OUR UNKNOWN

$$\frac{U^2}{gH} = F_H^2 = \left(\text{FROUDE NUMBER BASED ON DEPTH}\right)^2$$

SOLVE EQUATION:

$$F_H^2\, x = \tanh x$$

NOTE THAT $F_H < 1$ OCCURS IN ALL CASES OF SHIPS CRUISING IN CHANNES. FOR HIGH SPEED VESSELS $F_H > 1$ (BUT RARE).

LET x^* BE SOLUTION OF $F_H^2 x = \tanh x$

$$x^* = KH \Rightarrow K = \frac{x^*}{H}, \quad \omega = \frac{2\pi}{T} = KU = \frac{x^* U}{H}$$

$$\Rightarrow T = \frac{2\pi H}{x^* U} \cdot \underline{\quad}$$

b) THE SIGNAL DROPS TO ZERO WHEN THE SHIP WAVES BECOME SHORT ENOUGH FOR THEIR PENETRATION $(\lambda/2)$ TO BECOME EQUAL TO THE CHANNEL DEPTH H :

$$\frac{\lambda}{2} = H . \quad \text{(FOR SMALLER } \lambda \text{ WE DEAL}$$
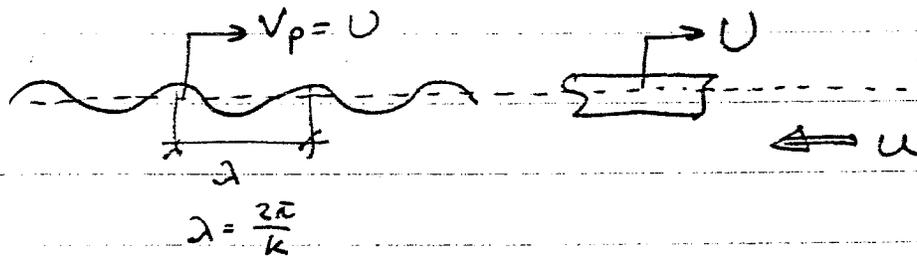$$\text{WITH DEEP WATER WAVES)}$$

$$2H = \lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega^2} g \qquad (\text{ASSUME DEEP WATER WAVES})$$

$$\Rightarrow \omega^2 = \frac{\pi g}{H} = \left(\frac{2\pi}{T^*}\right)^2$$

$$\Rightarrow \frac{\pi g}{H} = \frac{4\pi^2}{T^{*2}} \Rightarrow T^{*2} = \frac{4\pi H}{g}$$
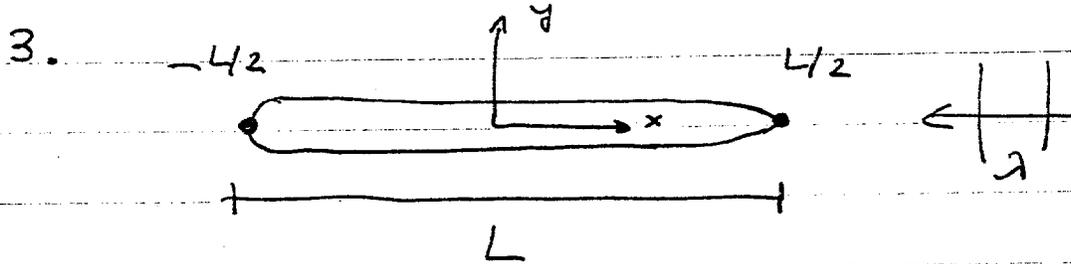
$$\Rightarrow T^* = \left(\frac{4\pi H}{g}\right)^{1/2} \cdot \underline{\hspace{1cm}}$$

c)

THE ADVERSE CURRENT u "CARRIES" THE WAVES
BACKWARDS THEREFORE REDUCIN THEIR PHASE
VELOCITY. TO $U-u$. THEIR WAVELENGTH λ
STAYS THE SAME SO THE NEW PERIOD RELATIVE
TO THE GAUGE (INERTIA FRAME IS)

$$T = \frac{\lambda}{U-u} = \frac{2\pi/k}{U-u}$$

WHERE k IS GIVEN BY: $\quad \frac{U^2}{g} k = \tanh kH \circ \underline{\hspace{1cm}}$

3.

FOR A SHIP WITH CONSTANT SECTION HYDRODYNAMICS THE PITCH EXCITING MOMENT IN HEAD WAVES IS GIVEN BY:

$$X_5 = \int_{-L/2}^{L/2} (-x) e^{i\nu x} \mathbf{C}$$

$\mathbf{C}$ = SUM OF SECTIONAL FROUDE KRYLOV + DIFFRACTION HYDRODYNAMIC FORCE ASSUMED CONSTANT.

• THE FACTOR WHICH DETERMINES THE AMPLITUDE OF THE EXCITING ~~FORCE~~ MOMENT IS :

$$\alpha = \int_{-L/2}^{L/2} x\, e^{i\nu x}\, dx = \int_{-L/2}^{L/2} x\,(\cos\nu x - i\sin\nu x)\, dx$$

BY VIRTUE OF SYMMETRY :

$$\alpha = -i \int_{-L/2}^{L/2} x \sin\nu x\, dx = -\frac{i}{\nu^3}\, 2 \int_{0}^{\nu L/2} \xi \sin\xi\, d\xi$$

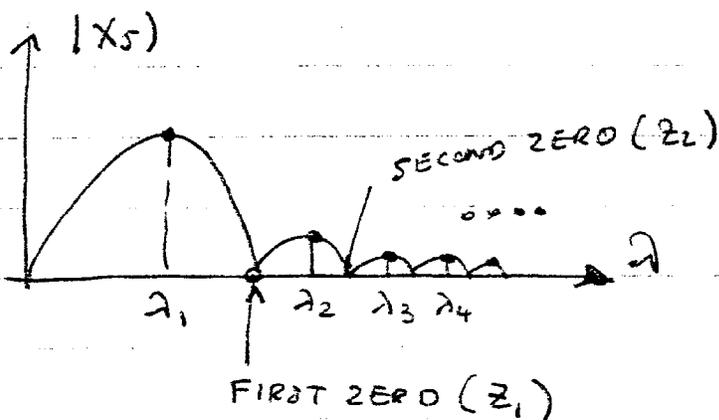$$\text{OR} \quad \alpha \sim \int_{0}^{\nu L/2} \xi \sin\xi\, d\xi = \sin\frac{\nu L}{2} - \frac{\nu L}{2}\cos\frac{\nu L}{2}$$

LET $\quad \dfrac{\nu L}{2} = z$, $\quad f(z) = \left| \sin z - z \cos z \right|$

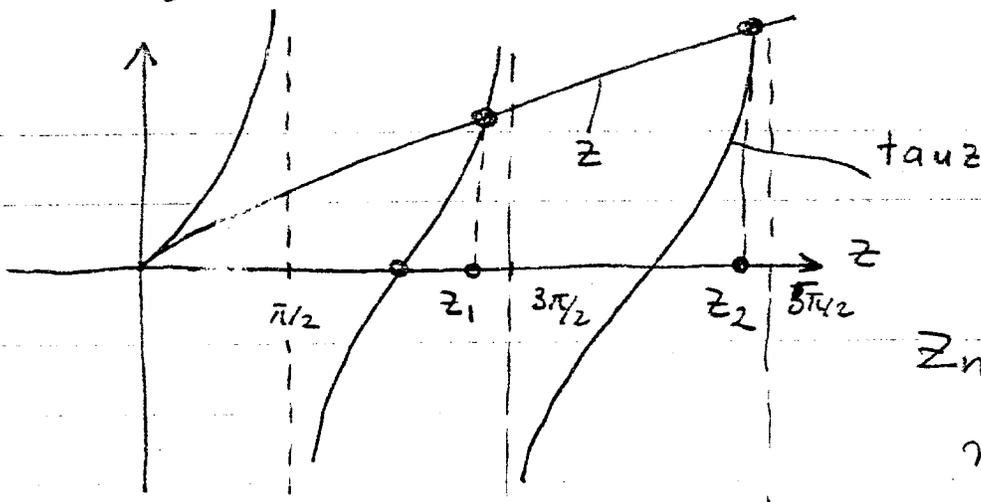$f'(z) = z \sin z = 0$, $\quad z = 0, \pi, 2\pi, 3\pi, = n\pi$

LOCATIONS OF MAXINA

$\dfrac{\nu L}{2} = n\pi$, $\quad n = 1, 2, \ldots$

$\Rightarrow \dfrac{2\pi L}{2\lambda} = n\pi \Rightarrow \lambda_n = \dfrac{L}{n}$

$f(z) = 0 \Rightarrow \sin z = z \cos z \Rightarrow z = \tan z$

$z_n \approx \dfrac{2n+1}{2} \pi$

$n = 1, 2, \ldots$

IN OBLIQUE WAVES:

$$X_s = \int_{-L/2}^{L/2} (-x) e^{-i\nu x \cos\beta} C(\beta)$$
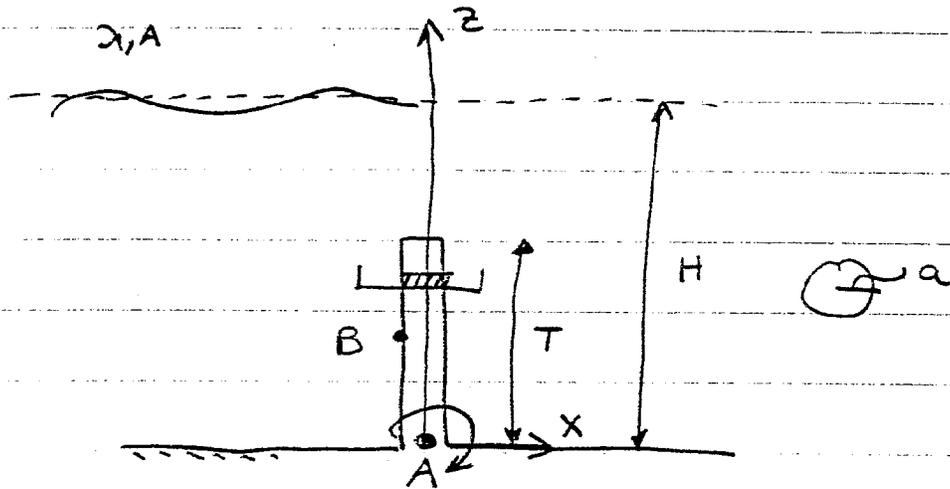
SO THE RELEVANT FACTOR NOW BECOMES:

$$\beta = \int_{-L/2}^{L/2} x e^{-i\nu x \cos\beta} dx$$

$$= \frac{1}{(\nu\cos\beta)^2} \int_{-L/2}^{L/2} (\nu x \cos\beta) e^{-i\nu x\cos\beta} d(\nu x\cos\beta)$$

$$\sim \int_{0}^{\frac{\nu L\cos\beta}{2}} \xi \sin\xi \, d\xi$$

SO THE ANSWERS FOLLOW FROM THE PREVIOUS QUESTION WITH $Z = \frac{\nu L\cos\beta}{2}$. —

WHEN $\beta = 90°$ THE PITCH MOMENT $X_s \equiv 0$!

d) THE ANSWERS IN $\alpha - c$ ARE LARGELY INDEPENDENT OF THE SPEED U SINCE THE DIFFRACTION PROBLEM FOR A SHIP WITH A PARALLEL MIDDLE BODY IS LARGELY INSENSITIVE TO THE FORWARD SPEED. —

4.

a)

$$\phi_I = \text{Re}\left\{ \frac{isA}{\omega} e^{\nu(z-H) - i\nu x + i\omega t} \right\}$$

● SURGE FORCE FOR A VERTICAL SLICE OF THICKNESS dz

$$dF_z = (a_{11} + \rho \forall) \frac{\partial^2 \phi_I}{\partial x \partial t} dz \qquad (G\,I\,TAYLOR)$$

● PITCH MOMENT ABOUT A:

$$X_5 = \int_0^T dz \; z \; dF_z$$

$$= \text{Re}\left\{ \frac{isA}{\omega}(i\omega)(-i\nu)(2\pi\rho a^2) \int_0^T z\, e^{\nu(z-H)}\, dz \right\}$$

$$= \text{Re}\left\{ i\omega^2 A \, 2\pi\rho a^2 \, e^{-\nu H} \int_0^T z\, e^{\nu z}\, dz \right.$$

$$\int_0^T z\, e^{\nu z}\, dz = \frac{1}{\nu^2} \int_0^{\nu T} \xi\, e^{\xi}\, d\xi = \frac{1}{\nu^2}\left[1 + (\nu T)e^{\nu T} - e^{\nu T} \right].$$

(ASSUME DEEP WATER WAVES FOR ALGEBRAIC SIMPLICITY)

b) THE LOCAL FLOW AT B CONSISTS OF A HORIZONTAL FLOW PAST A CIRCULAR SLICE AND A VERTICAL FLOW DUE TO THE AMBIENT WAVE WITH VERTICAL VARIATION $e^{\nu(z-H)}$.

LET ϕ_I BE THE INCIDENT AND ϕ_D THE DIFFRACTED WAVE DISTURBANCE. THE <u>INFLOW</u> VELOCITY AT B IS:

$$\vec{V_I} = \left(\frac{\partial \phi_I}{\partial x}, \frac{\partial \phi_I}{\partial z} \right) = (u_I, v_I)$$

THE INFLOW COMPONENT u IS DISTURBED BY THE TANK WHILE THE VERTICAL COMPONENT REMAINS LARGELY UNDISTURBED. THEREFORE WE WILL HAVE A DIFFRACTED VELOCITY COMPONEND FOR u

$$\vec{V}_{TOT} = (u_I + u_D, v_I)$$

THE DIFFRACTED VELOCITY u_D COMES FROM THE DISTURBANCE BY A CIRCLE OF AN INFLOW WITH VELOCITY u_I :
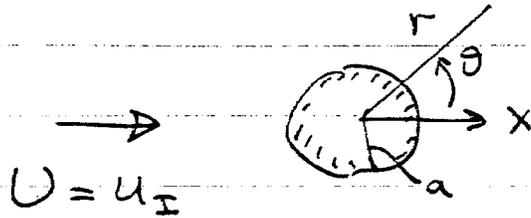
FLOW PAST A CIRCLE: $\phi_{TOT} = Ux - \dfrac{Ua^2}{r}\cos\vartheta$

$U_{TOT} = U - \dfrac{\partial}{\partial x}\left\{\dfrac{Ua^2}{r}\cos\vartheta\right\} \equiv U_I + U_D$

$U_D = -\dfrac{\partial}{\partial x}\left\{\dfrac{Ua^2}{r}\cos\vartheta\right\}$

THE LOCAL VERTICAL VARIATION OF THE FLOW IS:

$\phi_{TOT}(x,y,z) \sim e^{\nu(z-H)}\left\{Ux - \dfrac{Ua^2}{r}\cos\vartheta\right\}. -$

NEAR $x=0$ AND FOR $0 < z < T$.
(ASSUME DEEP WATER WAVES FOR SIMPLICITY)
THUS:
$e^{i\omega t}$

$\phi_{TOT} = \mathbb{Re}\left\{\dfrac{igA}{\omega}e^{\nu(z-H)}\left[Ux - \dfrac{Ua^2}{r}\cos\vartheta\right]\right\}$
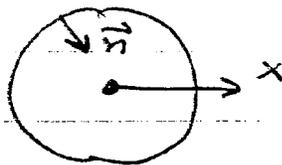
FOR x SMALL AND $0 < z < T. -$

LINEAR PRESSURE AT B: $p = -\rho\dfrac{\partial\phi_{TOT}}{\partial t}\Big|_{\substack{x=-a \\ z=T/2}} . -$

c) THE DEFINITION OF THE SURGE DRIFT FORCE IS:

$$\overline{F_x} = -\frac{\rho}{2} \int_{S_B} \overline{(\phi_x^2 + \phi_y^2 + \phi_z^2)}^{\,t} n_1 \, ds$$

WHERE ON THE CIRCLE $n_1 = \cos\vartheta$

$$\vec{n} = (n_1, n_2)$$
$$= (\cos\vartheta, \sin\vartheta)$$

THE VELOCITY POTENTIAL ϕ IS GIVEN BY THE EXPRESSION IN b):

$$\phi = \text{Re}\left\{ \frac{igA}{\omega} e^{\nu(z-H)} \left[U_I x - \frac{U_I a^2}{r}\cos\vartheta \right] e^{i\omega t} \right\}.$$

UPON SUBSTITUTION IN THE DEFINITION OF THE DRIFT FORCE THE EVALUATION IS POSSIBLE.

HOWEVER! SINCE THE LOCAL SECTIONAL FLOW IS A POTENTIAL FLOW PAST A CIRCLE THE TOTAL SECTIONAL FORCE (LINEAR + QUADRATIC) MUST BE ZERO BY VIRTUE OF D'ALAMBERT'S PARADOX. SO FOR THIS GEOMETRICAL CONFIGURATION AND FOR $\lambda \gg a$!

$$\overline{F_x} \simeq 0. \,-\!\!-$$

5/12/98
PDS