13.42 Homework #5

Spring 2005

Out: Thursday, March 3, 2005

Due: Tuesday, March 8, 2005

Problem 1: Consider the following discrete spectrum of unidirectional seas in deep water.

$$S_\eta(\omega) = \begin{cases} 8 & \text{if } \frac{1}{4} \le \omega < \frac{3}{4} \\ 12 & \text{if } \frac{3}{4} \le \omega < \frac{5}{4} \\ 10 & \text{if } \frac{5}{4} \le \omega < \frac{7}{4} \\ 4 & \text{if } \frac{7}{4} \le \omega < \frac{9}{4} \\ 2 & \text{if } \frac{9}{4} \le \omega < \frac{11}{4} \\ 0 & \text{otherwise} \end{cases}$$

a) Find the variance of the spectrum.
b) Find the average upcrossings of the sea state above the plane $z = 3\ m$.
c) Determine the minimum deck clearance, h, required for an offshore platform such that the deck floods less than once per hour. Assume no wave diffraction.

Problem 2: Given a continuous sea spectrum with a variance of 18 m^2 and a bandwidth of 0.6:

a) Find the probability of a wave maxima exceeding $5\ m$.
b) Find the probability of a wave maxima exceeding $10\ m$.
c) Find the required deck height of an offshore platform so that the deck has only a 1% chance of being flooded.

Problem 3: Consider a ship in heave with an input forcing function of:

$$f(t) = \sum_{i=1}^{N} f_i \cos(\omega_i t + \phi_i)$$

Where ϕ_i is a uniform distributed random variable from 0 to 2π. The input function, *f(t)*, is Gaussian with zero mean.

a) Write the equation of motion of the ship in heave.
b) Find the transfer function of the system.
c) Find the mean of the heave response.
d) Derive an expression for the variance of the heave in terms of the input spectrum, $S_f(\omega)$, *i=1,…,N.*
e) Find the mean of heave acceleration.
f) Derive an expression for the variance of the heave acceleration in terms of the input spectrum, $S_f(\omega)$, *i=1,…,N.*
g) Is heave acceleration a Gaussian process?