13.42 Homework #2

Spring 2005

Out: Thursday, February 10, 2005

Due: Thursday, February 17, 2005

<u>Problem 1</u>: There are two LTI systems: system 1 has transfer function $H_1(?\,)$, input $u_1(t)$ and output $y_1(t)$; and system 2 has transfer function $H_2(\,?\,)$, input $u_2(t)$ and output $y_2(t)$.

We set the input to system 1 to be a pure cosine:

$$u_1(t) = u_0 \cos(\boldsymbol{w}_0 t)$$

And the input to system 2 to be equal to the output of system 1:

$$u_2(t) = y_1(t)$$

Write down the output of system 2, $y_2(t)$.

<u>Problem 2</u>: Determine whether the following systems are LTI systems.

a) $\cos(\boldsymbol{w}_0 t) \rightarrow \boxed{\quad S \quad} \rightarrow 3\cos(3\boldsymbol{w}_0 t)$

b) $\sin(\boldsymbol{a}_0 t) \rightarrow \boxed{\quad S \quad} \rightarrow 3\sin(\boldsymbol{a}_0 t + \boldsymbol{y})$

<u>Problem 3</u>: Fourier Transform

a) Find the Fourier Transform of $f(t) = u_0(t - \boldsymbol{t})$.

b) Given that $f(x) \rightarrow 0$ as $|x| \rightarrow \infty$, and the Fourier Transform of $f(x)$ is $\tilde{f}(\boldsymbol{a})$, what is the Fourier Transform of $\dfrac{df}{dx}$? (Hint: Use partial integration.)

c) Given that $\dfrac{df}{dx} \rightarrow 0$ as $|x| \rightarrow \infty$, what is the Fourier Transform of $\dfrac{d^2 f}{dx^2}$?

Problem 4: Answer the following questions. You do not need to provide formal proofs in each case, but you should make sure that you understand the concepts underlying each result.

a) Given a stable, linear time invariant (LTI) system with transfer function $H(?\,)$, what is the steady state response (i.e., after the initial transients have died down) of the system when the input is sinusoidal with frequency $?$ and amplitude u_0?

b) You need to compute a transfer function to characterize the response of a model platform in waves. If you have access to a wave tank to do some experiments, can you find such a system? If so, describe briefly what kind of experiment(s) you would run and what calculations you would then need to make so that you could compute such a function.

Problem 5: Convolve the following inputs, $x(t)$, with $h(t)$ to find $y(t)$. Show the answer graphically, and explain your steps for part a) in detail:

a)

b)

c)
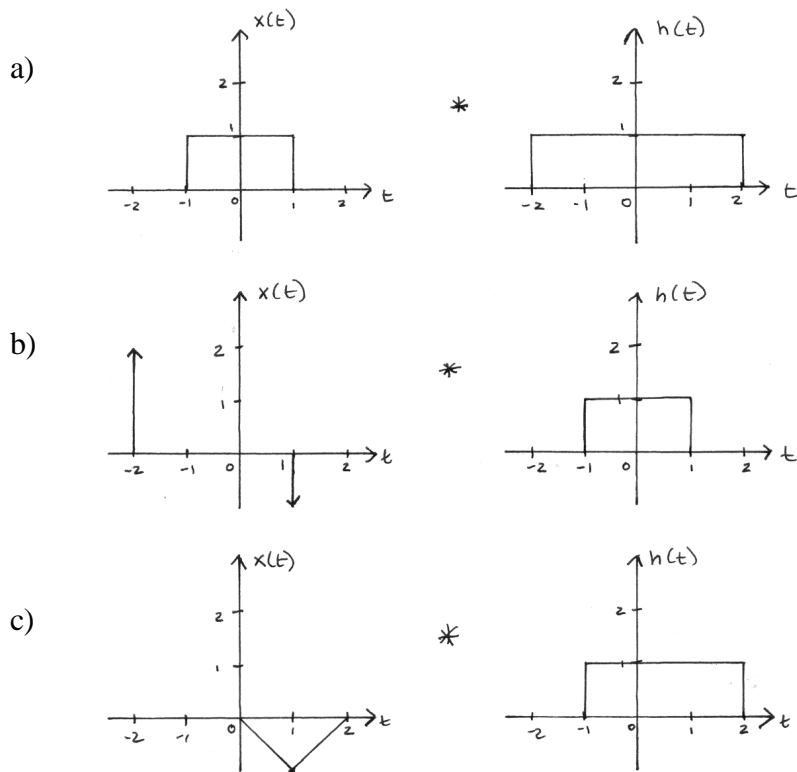
Problem 6: Fourier Transforms: Given a graph of $x(t)$, draw the graph of $\tilde{x}(w)$:
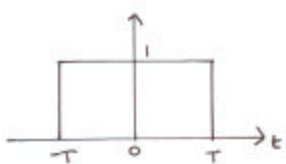
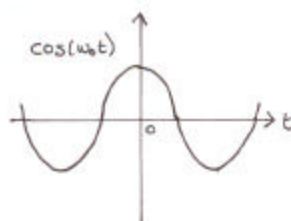$$x(t) \qquad\qquad \rightarrow \qquad\qquad \tilde{x}(w)$$

a)

b)

c)

Problem 7: Find the Fourier Transform of the following piecewise function:

$$x(t) = \begin{cases} 1 & \text{if } 1 \le x < 2 \\ 1.5 & \text{if } 2 \le x < 3 \\ 1 & \text{if } 3 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(Hint: Graph the function and decompose it into a combination of two simpler signals.)

<u>Problem 8</u>: Linear Waves: For designing ocean systems we need to understand when extreme environmental conditions occur. For linear free-surface gravity waves, the velocities and dynamic pressure varies harmonically in space and time.

a. Given a free surface elevation, $h(x,t) = a\cos(kx - wt)$, with wavelength, $l = 8\,m$, amplitude $a = 0.2m$, **non-dimensionalize and plot** the following quantities for a fixed point in space (i.e. fixed x position @ $x = x_o$ and fixed depth $z = z_n$) but varying time (the x-axis should be non-dimensional time and y-axis(or axes) should be the non-dimensional quantities being plotted):

$$h(x_o,t), \ u(x_o,t), \ w(x_o,t), \text{ and } p_d(x_o,t)$$

Choose the depth such that $z_n = 2\,n\,l$ (meters), where n is the integer corresponding to the month you were born in.

Line up the plots such that you can deduce the relative phase of the variables. You can plot them on one plot or multiple subplots – just make sure to label the different curves appropriately.

b. Plot the same quantities in part a, but with x varying and at one instant in time (i.e. @ $t = t_o$). The x-axis should be non-dimensional position in space and y-axis (or axes) should be the non-dimensional quantities being plotted.

c. At the depth determined by your birth month, what is the *total* pressure under the wave crest, the wave trough, the wave nodal point (the nodal point is the point at which the wave elevation corresponds to z = 0)?

d. The added mass force on a body is proportional to the fluid acceleration. Determine when the horizontal added mass force is maximum, minimum, and zero relative to the wave elevation (i.e. at the crest, the trough, or a nodal point).

e. While we are neglecting viscous forces for the most part in this course they are still important in certain applications. In general the non-linear, viscous wave forces are proportional to the square of the fluid velocity. When is the *horizontal* viscous wave force maximum – under the wave crest, the wave trough or the wave nodal point? When is the *vertical* viscous wave force maximum?