

2.20 Problem Set 2B

Name: _____

1. Complete the following statements:

- a) If you took a long time-exposure photograph of a luminous particle moving with the flow in a river, the image would be a (pathline , streakline , streamline).
- b) A _____ is a curve in a flow field which has the property that the velocity vector of each particle on the curve is tangent to the curve.
- c) There are a (finite , variable , infinite) number of streamlines in a flow field.
- d) If $d\vec{R}$ is the infinitesimal displacement vector along a streamline and \vec{V} is the velocity of a particle on the streamline, then $d\vec{R} \times \vec{V} = \underline{\hspace{2cm}}$.
- e) If you continuously inject dye at a fixed point into a flowing fluid in such a way that the dye particles essentially match the speed of the fluid as they enter the flow, the pattern of dye you see in a snapshot of the flow is a (pathline , streakline , streamline).
- f) A streamline is an (Eulerian , Lagrangian) concept.
- g) A pathline is an (Eulerian , Lagrangian) concept.

2. If you saw a plot of a flow in which two initially separate streamlines eventually intersected with each other at a point, what would that necessarily tell you about the fluid velocity at that point? Why?

3. We know that $\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$. Using vector identities, show that the following

expressing is equivalent: $\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - (\nabla \times \vec{V}) \times \vec{V}$. Note that $V^2 = \vec{V} \cdot \vec{V}$.

Hint: See Recitation 1 handout.

4. In a certain river with a velocity field $\vec{v}(\vec{x}, t)$ with respect to the ground, the concentration of dissolved oxygen is given by the function $f(\vec{x}, t)$. A small fish is in the river and has a velocity $\vec{U}(t)$ with respect to the ground.

The time rate of change of f experienced by the fish is:

The fish lays eggs that are very small and neutrally buoyant. The time rate of change of f experienced by the eggs is:

Some of the eggs eventually become trapped among rocks and remain there. The time rate of change of f experienced by the eggs is then:

5. In an incompressible flow where $\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + \vec{V} \cdot \vec{\nabla}\rho = 0$, the density ρ in general

- a) may vary in time at a particular point in the flow field
- b) must be a constant at all points in the flow field
- c) may vary in space but not time throughout the flow field

6. For a steady one-dimensional flow with a velocity magnitude u ,

- a) the velocity gradient $\partial u / \partial x$ must be zero at any point
- b) the density ρ must be the same at all points in space
- c) the rate of change of the density of a particle as it moves through the flow depends only on the spatial variation of the density