2.20 Problem Set #1 **Name: _____**

Purpose: This is a multiple choice/short answer type of exercise prepared to give the teaching staff a "snapshot" of the level of knowledge of the class at the start. No number grade will be assigned, but a note will be made of whether you did it or not.

Rules for Completion: This is a closed book exercise. No collaboration is allowed with others – this is a test of your knowledge. Try to do the exercise without interruption. When you are done, please note below roughly how long you actually spent on the exercise.

Approximate Time to Complete: _____ minutes (to nearest 15 minutes)

For the following, circle the letter of the correct answer or fill in the blank:

1. Which of the following would describe a constitutive relation for a material?

a) As pressure increases, temperature increases
b) As volume decreases, pressure increases
c) As the rate of stretching increases, tensile stress increases

2. In a conservative mechanical system, external forces can be represented as

a) the curl of a vector potential
b) the gradient of a scalar potential
c) the divergence of a vector potential

3. When we state for a system $\vec{F} = m\vec{a}$, it is implied that the frame of reference is

a) fixed in space
b) accelerating with respect to a fixed frame
c) either fixed or translating at a constant velocity with respect to a fixed frame

4. The following equation gives the acceleration of a point P with respect to a fixed frame OXYZ using an intermediate frame oxyz:

$$\vec{a} = \frac{d^2\vec{R}_o}{dt^2} + \vec{a}_{rel} + 2\vec{\omega}\times\vec{v}_{rel} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$

where $\vec{R}_o$ is the position of the origin of oxyz with respect to OXYZ, $\vec{v}_{rel}$ is the velocity of P with respect to oxyz, $\vec{a}_{rel}$ is the acceleration of P with respect to oxyz, and $\vec{\omega}$ is the angular velocity of oxyz with respect to OXYZ.

The Coriolis acceleration term is _____ . The centripetal acceleration term is _____ .

5. For a system of particles, the linear impulse is equal to

a) the difference between the initial and final kinetic energy
b) the difference between the initial and final linear momentum
c) the product of the total mass and acceleration of the center of gravity

6. The natural frequency of an undamped spring-mass system having mass m and spring constant k is

a) $\sqrt{k/m}$
b) $\sqrt{km}$
c) $\sqrt{m/k}$
d) none of the above

7. The moment of inertia of a solid sphere of mass m and radius r about any centroidal axis is

a) $(1/2)mr^2$
b) $(2/5)mr^2$
c) mr^2

8. The angular momentum of a object with a moment of inertia *I* about the spin axis and an angular velocity ω is _____ .

9. The angular momentum principle

$$\vec{\tau}_B = \frac{d\vec{H}_B}{dt}$$

where $\vec{\tau}_B$ is the total external torque acting on the rigid body and $\vec{H}_B$ is the angular momentum about a moment center B is valid when

a) The point B has zero velocity with respect to the inertial reference frame\
b) The point B has a velocity parallel to the velocity of the body's centroid
c) a) or b)
d) never

10. The work done on a particle by all forces acting on the particle is equal to the change of _____ of the particle.

11. Which of the following is not conserved in a fluid system?

a) mass
b) pressure
c) energy
d) momentum

12. The assumption of a material continuum would be appropriate for the following fluid flows:

a) water in a pipe
b) air flowing through a fan
c) lava flowing down a mountainside
d) all of the above
e) a) and b) only

13. Which of the following is not in general a characteristic of a flow with velocity due to a velocity potential ?

a) $\nabla^2 \phi = 0$
b) $\vec{V} = \vec{\nabla}\phi$
c) $\vec{V} = \vec{0}$ on any solid boundary of the fluid flow

14. The Reynolds number represents the ratio of the following forces in a fluid flow:

a) inertia force to gravity force
b) pressure force to inertia force
c) inertia force to viscous force
d) inertia force to surface tension force

15. The symmetry of the fluid stress tensor results from all of the following except

a) conservation of linear momentum
b) conservation of angular momentum
c) the continuum hypothesis
d) conservation of energy

16. In a Newtonian fluid, the stress due to fluid motion is

a) independent of strain rate
b) a linear function of strain rate
c) a quadratic function of strain rate

17. If energy input to a viscous flow ceases, then any existing vorticity must

a) remain constant since angular momentum is conserved
b) eventually decay to zero
c) be converted into kinetic energy

18. In an incompressible flow where $D\rho/Dt = 0$, the density ρ in general

a) may vary in time at a particular point in the flow field
b) must be a constant at all points in the flow field
c) may vary in space but not in time throughout the flow field

19. The Bernoulli equation for unsteady potential flow applies

a) only along a streamline of the flow
b) at any point in the fluid domain
c) only along the boundary of the fluid domain

20. A boundary layer is formed near the surface of a moving submarine due to

a) density variation
b) viscosity
c) surface tension
d) pressure variation

For the following, show any steps taken in obtaining your answer:

21. Evaluate $\int_{-2}^{0} x\sqrt{2x^2+1}\,dx$

22. Evaluate $\int \sin^3 x\,dx$

23. Evaluate $\frac{d}{dx}\left(e^{3x}\ln|x|\right)$

24. Consider a function $\vec{V}=\vec{V}\left(x(t),y(t),z(t),t\right)$. Write an expression for the total derivative $d\vec{V}/dt$.

25. Evaluate $\int_0^{\pi/2} x\cos x dx$

26. For a function $f=r^2\cos\theta$, evaluate $\vec{\nabla}f$ in polar coordinates:

27. For fluid velocity $\vec{V}=(u,v,w)$, expand $\vec{\nabla}\times\vec{V}$ in rectangular coordinates:

This quantity is known as fluid _____ .

28. Find the general solution of the differential equation: $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$

29. Solve the following differential equation:

$$\frac{dy}{dx}+\frac{1}{x}y=3x \qquad\qquad y(1)=0$$

30. Complete the expression on the right hand side: $\iiint_V \vec{\nabla}\cdot\vec{F}dV=\iint_S \_\_\_\_dS$