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2.161 Signal Processing: Continuous and Discrete  
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF MECHANICAL ENGINEERING

*2.161 Signal Processing - Continuous and Discrete*  
Fall Term, 2008

**Quiz #1 — October 16, 2008**

Notes:

- The quiz is closed-book, two sheets of notes are allowed.
- The time allowed is ninety minutes.
- There are 4 problems, answer them all.
- Partial credit will be given.
- **Some possibly useful information is given at the back of this quiz.**

Problem 1: (20 points)

A low-pass filter is to be designed from the power frequency response function

$$|H(j\Omega)|^2 = \frac{1}{1 + \Omega^4}$$

- (a) Determine a stable system transfer function based on this specification.
- (b) Make a pole-zero plot for the system and determine the filter class (low-pass, etc).
- (c) What type of design is this (Butterworth, Chebyshev Type I, etc).
- (d) Transform your filter design to a new filter  $H_1(s)$ , using the substitution

$$\frac{2s}{s^2 + 4} \longrightarrow s$$

in  $H(s)$ .

- (e) How many poles and zeros does  $H_1(s)$  have, and what class of filter (low-pass etc) is it?.

Problem 2: (20 points)

Your advisor (who should remain nameless!) went out and bought you a new lab computer complete with A/D converters that will sample up to 10,000 samples/second. Without consulting you he also went and bought a pressure sensor that produces a sinusoidal output with a frequency that is proportional to pressure, in the range 0 to 7 kHz. He expects you to use the computer with the sensor.

Your heart skips a beat. “Oh, no – it cannot be!”, you cry in despair, “The output of this sensor exceeds the maximum Nyquist rate allowed by my computer!”

But, in fact, is everything lost?

Assume that you have software that will allow you to take a record of sinusoidal data with a sampling interval  $\Delta T$ , and then report back its frequency. Determine and describe an experimental procedure that will allow you to determine the frequency of the sensor output in the range 0 – 7 kHz, using a maximum sampling rate of 10 kHz.

**Hints:** 1) Remember that you are not asked to reconstruct an arbitrary unknown waveform, therefore you know the effect of aliasing.

2) You may want to consider taking more than one data record with different sampling rates.

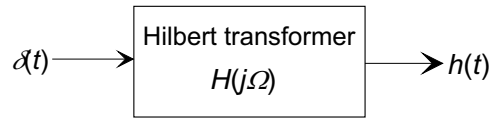
Problem 3: (35 points)

We are going to design a simple *speech scrambler*, that will encode an audio signal for “secure” transmission (that is make it unintelligible) over the internet, yet allow it to be decoded at the receiving end by a similar process.

Let’s start by investigating a *Hilbert transformer*, which is a linear system with a frequency response function

$$H(j\Omega) = \begin{cases} -j, & \Omega > 0 \\ j, & \Omega < 0, \end{cases}$$

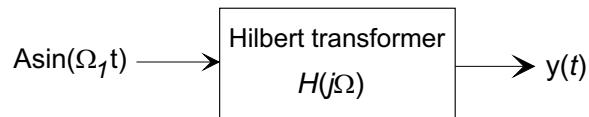
- (a) Determine the impulse response  $h(t)$  of the Hilbert transformer.



Is this a causal filter?

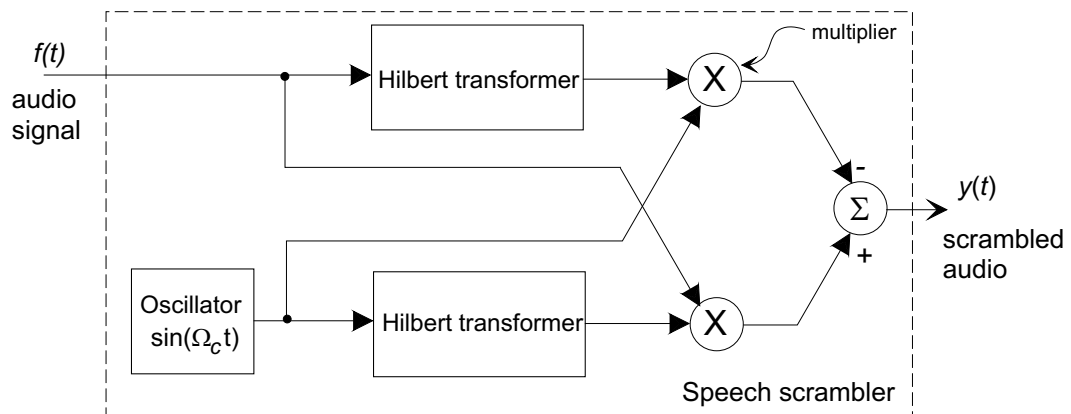
**Hint:**  $\mathcal{F}\{\text{sgn}(t)\} = 2/(j\Omega)$  where  $\text{sgn}()$  is the signum function.

- (b) If the input to the Hilbert transformer is a sinusoid  $A \sin(\Omega_1 t)$ , what is the output  $y(t)$ ?



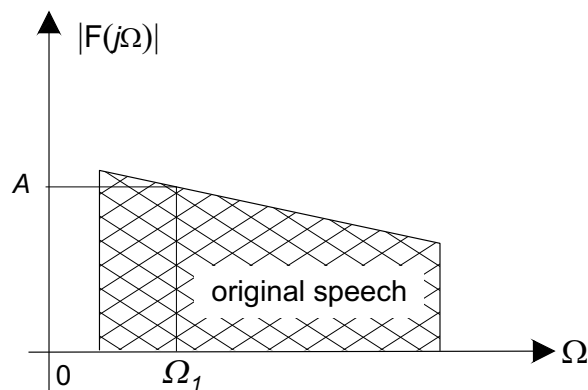
What is the general effect on sinusoidal signals?

- (c) Now consider the block diagram of the complete speech scrambler below:



Internally an oscillator generates a constant frequency sinusoid  $\sin(\Omega_c t)$ . Assume for now that the audio signal is a sinusoidal waveform  $A \sin(\Omega_1 t)$ . Determine the output of the system  $y(t)$ .

- (d) If the audio signal has a (one-sided) spectrum shown below



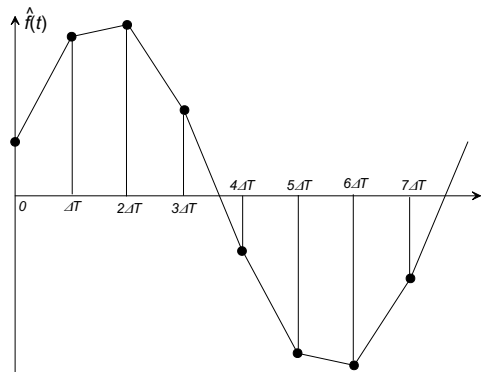
sketch the magnitude spectrum  $|Y(j\Omega)|$  of the output signal  $y(t)$ . What will be the amplitude and frequency of the component shown with frequency  $\Omega_1$ ? Explain how the scrambling operation works.

- (e) For extra credit, suggest what the “descrambler” needs to do to restore the original signal.

#### Problem 4: (25 points)

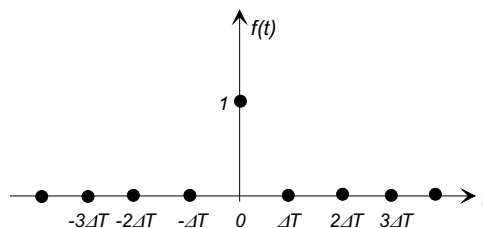
A linear interpolator is a hardware, non-ideal, data reconstructor in which successive samples are connected by straight line segments (“connect-the-dots”). If the sample set is  $\{f(n\Delta T)\}$ , the reconstructed continuous waveform  $\hat{f}(t)$  can be written

$$\hat{f}(t) = f((n-1)\Delta T) + \frac{f(n\Delta T) - f((n-1)\Delta T)}{\Delta T}(t - n\Delta T), \quad n\Delta T \leq t < (n+1)\Delta T$$



Notice that when  $t = n\Delta T$ , the interpolator output  $\hat{f}(t) = f((n-1)\Delta T)$ , that is it is one step  $\Delta T$  behind. If we treat the linear interpolator as a filter,

- (a) Consider the response of the interpolator to the digital pulse train below:



Show that the linear interpolator (with a single-step delay as above) has an pulse response

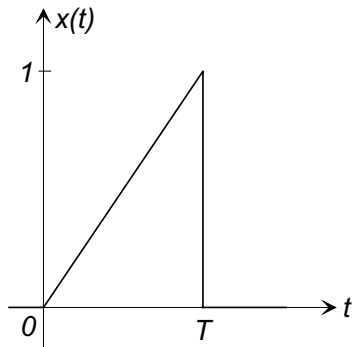
$$\hat{h}(t) = \begin{cases} t/\Delta T, & 0 \leq t < \Delta T \\ 2 - t/\Delta T, & \Delta T \leq t < 2\Delta T, \\ 0, & \text{otherwise.} \end{cases}$$

Sketch the pulse response (to help you in the next part).

- (b) Find the frequency response function  $\hat{H}(j\Omega)$  of the linear interpolator. You can use any method but, consider the two following possible methods:

**Method 1:**

The waveform  $x(t)$  to the left

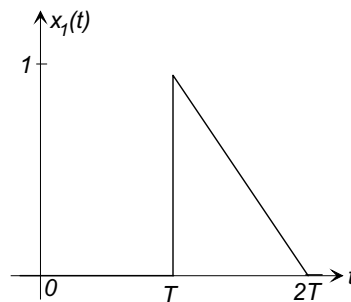


$$x(t) = \begin{cases} t/T, & 0 \leq t < T \\ 0, & \text{otherwise.} \end{cases}$$

has a Fourier transform

$$X(j\Omega) = \frac{e^{-j\Omega T}(j\Omega T + 1) - 1}{\Omega^2 T}$$

- (i) Use  $X(j\Omega)$  to find the Fourier transform of the waveform  $x_1(t)$  below



- (ii) Combine these results to find  $\hat{H}(j\Omega)$ .

**Method 2:**

- Consider  $h(t)$  as the convolution of two  $\text{rect}()$  functions...

- (c) Make a sketch of  $|\hat{H}(j\Omega)|$ , and compare it to the frequency response of the ideal (cardinal) reconstructor.

Some useful/useless information:

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0	1	0
$\pi/6$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	1/2	$\sqrt{3}$
$\pi/2$	1	0	$\infty$

$x$	$\log_e(x)$	$\log_{10}(x)$
2	0.6931	0.3010
3	1.0986	0.4771
5	1.6094	0.6990
10	2.3026	1.0000

$$\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)}$$

$$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a) \tan(b)}$$