MULTI-DOMAIN MODELING

WHAT'S THE ISSUE?

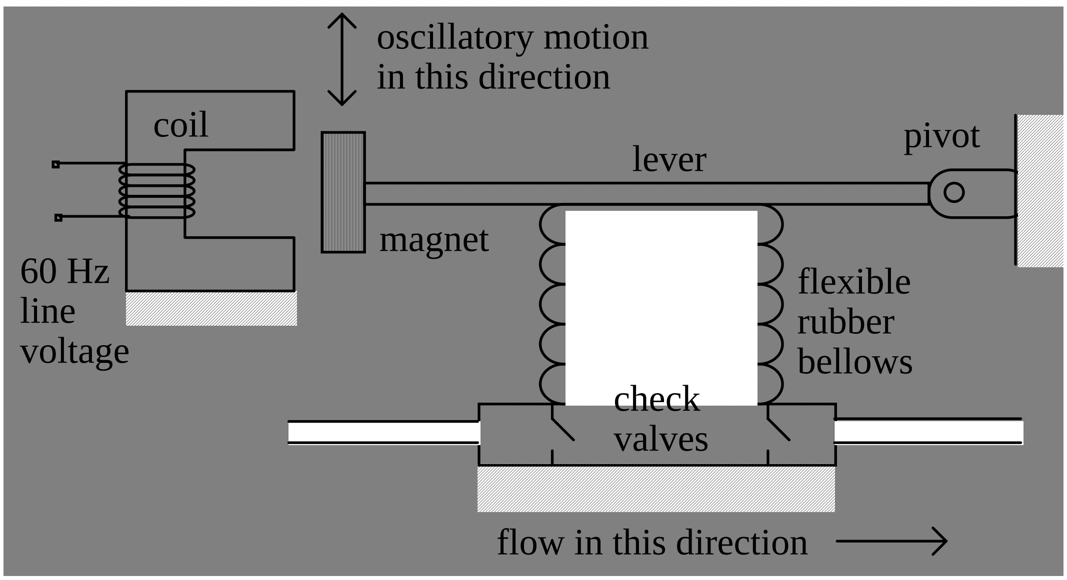
Why not just "write down the equations"?

— standard formulations in different domains are often incompatible

usually due to incompatible boundary conditions (choice of "inputs")

EXAMPLE: SIMPLE FLUID SYSTEM

Scenario



A typical low-cost aquarium air-pump

use "resonant" air pump design as a low-cost fuel pump

does the pump really resonate?

would it do so with fuel not air?

Modeling goal

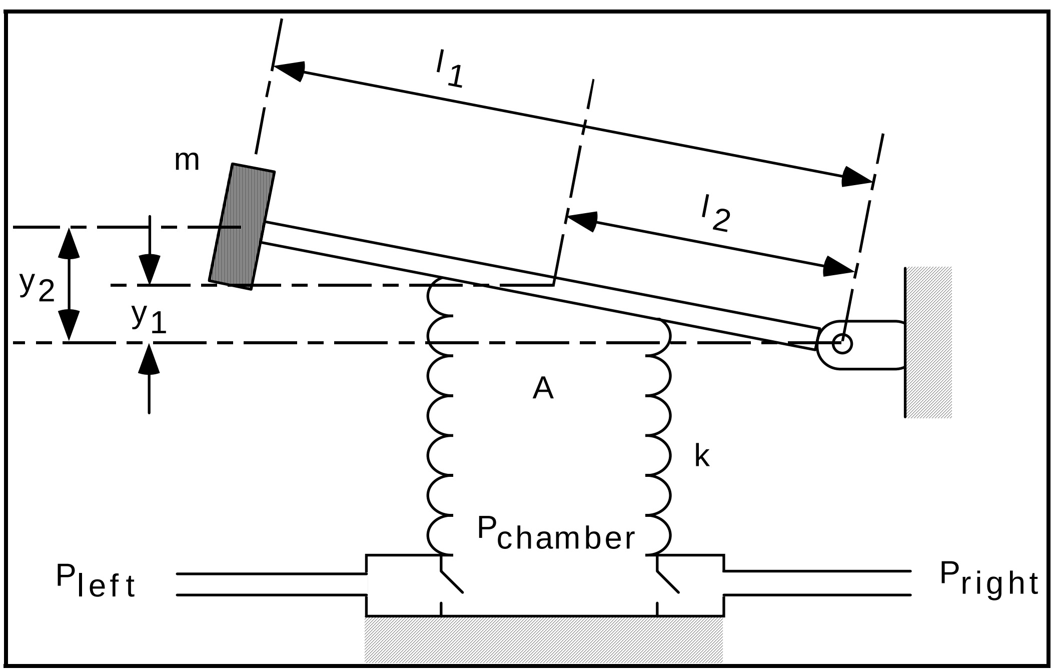
The simplest model competent to portray resonance in this system.

Assumptions

- rigid-body motion (e.g., lever & pivot do not bend)
- mass concentrated at c.g. of magnet
- electro-magnet applies an oscillating force
- small motion
- mechanical elasticity only in the flexible bellows
- no power dissipation except in the check valves (this assumption highlights any resonant behavior)
- linear model of mechanical-to-fluid transduction
- incompressible fluid
- ideal check valves

"Write down" the equations

First define quantities:



Mechanical Domain

$m\,\ddot{y}_2 := F_{magnet} - F_{spring} - F_{dissipation}$

(symbol := denotes an *assignment operator;* right side is computed to evaluate left side.)

$$F_{spring} := k\,\frac{l_1}{l_2}\;y_2$$

$F_{dissipation}$ is due to fluid forces. Substituting:

$$m\,\ddot{y}_2 := F_{magnet} - \frac{l_1}{l_2}\;k\,\frac{l_1}{l_2}\;y_2 - \frac{l_1}{l_2}\;F_{dissipation}$$

This implies state equations:

$$\frac{d}{dt}\;y_2 := \dot{y}_2$$

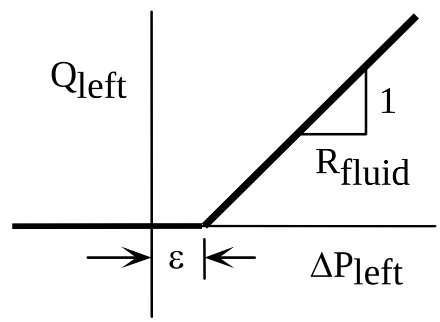
$$\frac{d}{dt}\dot{y}_2 := \frac{1}{m}\left[F_{magnet} - \frac{l_1}{l_2}k\frac{l_1}{l_2}y_2 - \frac{l_1}{l_2}F_{dissipation}\right]$$

Fluid Domain

Left check valve:

$\Delta P_{left} := P_{left} - P_{chamber}$

$$Q_{left} := \begin{cases} 0 \text{ if } \Delta P_{left} \le \varepsilon \\ \dfrac{(\Delta P_{left} - \varepsilon)}{R_{fluid}} \text{ if } \Delta P_{left} > \varepsilon \end{cases} := g(\Delta P_{left})$$



where ε is a small constant

—flow is blocked until pressure is sufficient to overcome a preload

Right check valve similar

Couple the domains

Relation: pressure = force times area

$F_{dissipation} := A\,P_{chamber}$

SNAG!	$P_{chamber}$ is a fluid domain **input**, not **output**.

Try manipulating the check-valve equations:

$P_{chamber} := P_{left} - \Delta P_{left}$

But $\Delta P_{left} \neq f(Q_{left})$

That is, the function $\Delta P_{left} := f(Q_{left})$ cannot be defined because the nonlinear function $Q_{left} = g(\Delta P_{left})$ has no inverse.

Try manipulating the mechanical equations:

If $P_{chamber}$ is a fluid domain input, $Q_{chamber}$ is a fluid domain output.

$$\dot{y}_1 := \frac{1}{A} Q_{chamber}$$

$$\dot{y}_2 := \frac{l_2}{l_1} \dot{y}_1$$

$\frac{d}{dt} y_2 := \dot{y}_2)$ (a differential equation for state variable y_2 as before).

$$Q_{chamber} := Q_{left} - Q_{right}$$

$$Q_{left} := g(P_{left} - P_{chamber})$$

$$Q_{right} := g(P_{chamber} - P_{right})$$

$$P_{chamber} := \frac{1}{A} F_{dissipation}$$

$$F_{dissipation} := \frac{l_2}{l_1} F_{magnet} - k \frac{l_1}{l_2} y_2 - \frac{l_2}{l_1} m \ddot{y}_2$$

But **NOTE** that $\dot{y}_2$ is **not** a state variable as before.

Incompetent model!

In fact, this is a *first-order* system of equations and hence is *incapable* of portraying resonance in the system. "Pseudo-code" for the sorted equations are as follows.

$$\frac{d}{dt}\, y_2 := \dot{y}_2$$

$$\dot{y}_2 := \frac{l_2}{l_1}\dot{y}_1$$

$$\dot{y}_1 := \frac{1}{A}\, Q_{chamber}$$

$$Q_{chamber} := Q_{left} - Q_{right}$$

$$Q_{left} := \text{IF}$$

$$Q_{left} := \text{IF } \Delta P_{left} > \varepsilon \text{ THEN } (\Delta P_{left} - \varepsilon)/R_{fluid} \text{ ELSE } 0$$

$$Q_{right} := \text{IF } \Delta P_{right} > \varepsilon \text{ THEN } (\Delta P_{right} - \varepsilon)/R_{fluid} \text{ ELSE } 0$$

$$\Delta P_{left} := P_{left} - P_{chamber}$$

$$\Delta P_{right} := P_{chamber} - P_{right}$$

$$P_{chamber} := \frac{1}{A}\, F_{dissipation}$$

$$F_{dissipation} := \frac{l_2}{l_1}\, F_{magnet} - k\frac{l_1}{l_2}\, y_2 - \frac{l_2}{l_1}\, m\, \ddot{y}_2$$

Note that only one constant of integration (i.e., initial condition) is required for the solution.

$$y_2(t) := \int_0^t \dot{y}_2\, dt + y_2(0)$$

Note also that a derivative operation is required.

$$\ddot{y}_2(t) := \frac{d}{dt}\dot{y}_2(t)$$

DISCUSSION

The model is well formulated

The wisdom of the stated assumptions might be debated, but they are entirely consistent with the stated goal, to formulate the simplest model competent to portray resonant behavior.

— the standard equations are not

The "standard equations" for rigid-body mechanical motion implicitly assume that "force causes motion" — i.e., force is always an *input* to mass elements and their resulting motion is the *output*. But Newtonian physics only specifies a *relation* between force and acceleration, not an assignment of input and output.

Similarly, the "standard equations" in the fluid domain assume that pressure gradient cause flows, but physics only specifies a relation.

— the problem is due to boundary conditions

The standard form for the mechanical sub-system in incompatible with the assumed fluid sub-system.

"Just writing down" the equations of motion usually works for a single domain but often fails for multi-domain systems.

"Standard" formulations in different domains implicitly assume specific "boundary conditions" (i.e., a particular choice of "inputs" and "outputs"). Unfortunately, these boundary conditions are often incompatible—as above.

Network modeling approach

Analysis of the relations between model domains prior to equation derivation reveals incompatibilities.

clarifies boundary conditions

identifies origins of conflicts

defers writing equations until needed