ELECTRO-MAGNETIC INTERACTION

The relation between electricity and magnetism is another example of intimate interaction between energy domains.

NETWORK MODELS OF MAGNETISM

Physics:

A magnetic field embodies a relation between flux density, B, and magnetic field strength (or intensity) H.

In air or in vacuum, the relation is linear.

$$B = \mu_o H$$

where μ_o is the permeability of air or vacuum.

In a ferro-magnetic material such as soft iron the relation may be nonlinear.

$$B = B(H)$$

That relation is frequently linearized as follows.

$$B = \mu_r \mu_o H$$

where μ_r is the relative permeability of the material.

Network variables:

Magnetomotive force, F:

$$F = H\,l$$

where l is the length of the magnetic path.

Magnetic flux, φ:

$$\varphi = B\,A$$

where A is the cross-sectional area of the magnetic path.

In the linear case:

$$F = (l/\mu_r\mu_o A)\varphi = R\,\varphi$$

where R is the *reluctance* of the magnetic path.

THE FLAW IN THE CONVENTIONAL ANALOGY

— REPRESENT A MAGNETIC PATH AS AN ELECTRICAL RESISTOR

magnetomotive force	$\leftrightarrow$	electromotive force (voltage)
magnetic flux	$\leftrightarrow$	electrical current
magnetic reluctance	$\leftrightarrow$	electrical resistance

A SERIOUS FLAW:

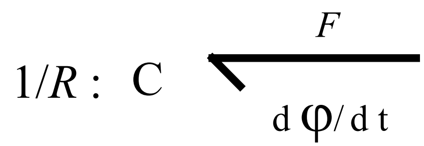
A magnetic field *stores* energy

An electrical resistor *dissipates* energy

(to be precise, a resistor dissipates *free* energy — more on this later)

AN ENERGETICALLY CONSISTENT ANALOGY

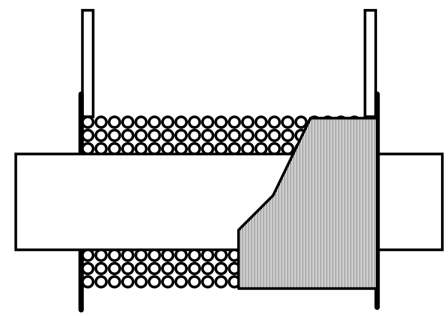
— REPRESENT A MAGNETIC PATH AS A CAPACITOR



(inverse) capacitance	↔	magnetic reluctance
effort	↔	magnetomotive force
displacement	↔	magnetic flux
flow	↔	magnetic flux rate

NETWORK MODELS OF ELECTRO-MAGNETISM

A magnetic field may be generated by a coil of wire carrying a current.



Magnetomotive force

$$\mathcal{F} = N\, i$$

where N is the number of turns of wire in the coil.

Magnetomotive force is an effort variable

electrical current is a flow variable.

—one constitutive equation of a gyrator

The amount of flux "linked" by the coil is the *flux linkage*, λ

$$\lambda = N\varphi$$

Faraday's law:

The rate of change of flux linkage determines the voltage across the coil.

$$\frac{d\lambda}{dt} = N\frac{d\varphi}{dt}$$

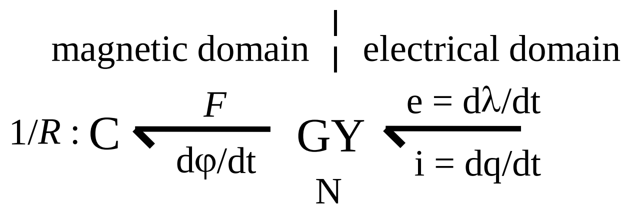
$$e = N\,\dot{\varphi}$$

—the other constitutive equation of the gyrator

ELECTRO-MAGNETIC COUPLING MAY BE REPRESENTED AS A GYRATOR

ELECTRICAL INDUCTOR

A coil of wire wrapped on a ferro-magnetic core may be modeled by a (magnetic) capacitor and a gyrator.

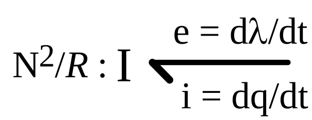


Bond graph of a simple electromagnetic coil model.

It often serves as an electrical circuit component.

Equivalent electrical behavior

—an inductor.



Assuming magnetic linearity, the electrical constitutive equation is

$$\lambda = \frac{N^2}{\mathcal{R}}\ i = L\ i$$

where $L = N^2/\mathcal{R}$ is the inductance of the coil.

Differentiating results in a more familiar form.

$$e = L\ di/dt$$

ELECTRICAL INDUCTOR BEHAVIOR IS

MAGNETIC CAPACITOR BEHAVIOR

TRANSDUCED THROUGH A GYRATOR.